LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Dr. Iosif Petrakis

Winter term 19/20
11.11.2019

Mathematics for Natural Scientists I

Sheet 5

Exercise 1. Let $(\alpha_n)_{n\in\mathbb{N}}$ be sequence of reals, and $\lambda, x \in \mathbb{R}$. Let the sequence $(\lambda\alpha)_{n\in\mathbb{N}}$, defined by

$$(\lambda\alpha)_n = \lambda\alpha_n,$$

for every $n \in \mathbb{N}$. If $\alpha_n \xrightarrow{n} x$, show that $(\lambda\alpha)_n \xrightarrow{n} \lambda x$.
[4 points]

Exercise 2. **(i)** Let the sequence α of reals defined by

$$\alpha_n = \frac{4n^2 + 14n}{n^3 - 2},$$

for every $n \in \mathbb{N}$. Find the limit of $(\alpha_n)_{n\in\mathbb{N}}$.
[2 points]
(i) Let the sequence β of reals defined by

$$\beta_n = \frac{4n^3 + 14n}{n^2 - 2},$$

for every $n \in \mathbb{N}$. Explain why $(\beta_n)_{n\in\mathbb{N}}$ is divergent.
[2 points]

Exercise 3. Let $(\alpha_n)_{n\in\mathbb{N}}$ be a sequence of reals, and $x, A, B \in \mathbb{R}$. If $\alpha_n \xrightarrow{n} x$, and if

$$A \leq \alpha_n \leq B,$$

for every $n \in \mathbb{N}$, show that $A \leq x \leq B$.
[4 points]

Exercise 4. Show that there is no rational number q such that $q^2 = 3$.
[4 points]

Submission. Wednesday 20. November 2019, in the Exercise-session.

Discussion. Wednesday 20. November 2019, in the Exercise-session.