Dr. Iosif Petrakis

Winter term 19/20
28.10.2019

Mathematics for Natural Scientists I

Sheet 3

Exercise 1. **(i)** Show that the number 1 in the axiom (M_2) is unique.
[1 point]
(ii) Show that the number y in the axiom (M_3) is unique.
[1 point]
(iii) With the use of the axioms $(M_1) - (M_4)$ show that for every $x, y \in \mathbb{R}$ the following implication holds:

$$[x \neq 0 \ \& \ y \neq 0] \Rightarrow xy \neq 0.$$

[1 point]
(iv) With the use of the axioms (O_1) and (O_2) show that for every $x, y \in \mathbb{R}$ the following implication holds:

$$xy = 0 \Rightarrow [x = 0 \ \vee \ y = 0].$$

[1 point]

Exercise 2. If $x, y, z, w \in \mathbb{R}$, show the following.
(i) $-(x + y) = -x - y$.
[1 point]
(ii) If $z, w \neq 0$, then

$$\frac{x}{z}\frac{y}{w} = \frac{xy}{zw} \quad \& \quad \frac{x}{z} + \frac{y}{w} = \frac{xw + yz}{zw}.$$

[1 point]
(iii) If $x \neq 0$ and $xy = xz$, then $y = z$.
[1 point]
(iv) Using the axioms (I), (II) and (III) show that $(x + y)^2 = x^2 + 2xy + y^2$.
[1 point]

Exercise 3. If $x, y, z \in \mathbb{R}$, show the following.
(i) If $x > 0$, then $\frac{1}{x} > 0$.
[1 point]
(ii) If $x < y$ and $y < z$, then $x < z$.
[1 point]
(iii) If $x < y$ and $z > 0$, then $xz < yz$.
[1 point]
(iv) If $x < y$ and $x, y > 0$, then $\frac{1}{y} < \frac{1}{x}$.
[1 point]

Exercise 4. Let $x, y \in \mathbb{R}$.
(i) If $x, y \geq 0$ and $\sqrt{x}, \sqrt{y}$ exist, show that $\sqrt{xy}$ exists, that

$$\sqrt{xy} = \sqrt{x}\sqrt{y},$$

[1 point]
and that

$$x \leq y \Rightarrow \sqrt{x} \leq \sqrt{y}.$$

[1 point]
(ii) Using the triangle inequality show that

$$\big||x| - |y|\big| \leq |x - y| \leq |x| + |y|.$$

[2 points]

Submission. Wednesday 06. November 2019, in the Exercise-session.

Discussion. Wednesday 06. November 2019, in the Exercise-session.