Dr. Iosif Petrakis

Winter term 19/20
29.01.2020

# Mathematics for Natural Scientists I

Sheet 15
Probeklausur

**Exercise 1.** Show that for every $n \geq 1$ the following equality holds

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}.$$

**[6 points]**

**Exercise 2.** **(i)** Let the following sequence of real numbers

$$\alpha_n = \frac{2020n^5 + 2019n^3 + 2018}{n^5 + 2019n^2 + 2020}, \qquad n \in \mathbb{N}.$$

Find the limit

$$\lim_{n \longrightarrow \infty} \alpha_n.$$

**[2 points]**

**(ii)** Calculate the following real number:

$$\sum_{n=1}^{\infty} \frac{5}{3^n}.$$

**[2 points]**

**(iii)** Give an example of an irrational number in the interval $(0, 1)$.

**[2 points]**

**Exercise 3.** **(i)** Let the functions $h_1, h_2, h_3 : \mathbb{R} \to \mathbb{R}$ defined by

$$h_1(x) = x^2 - 2,$$

$$h_2(x) = 2 - x^2,$$

$$h_3(x) = (x-1)^2 - 2,$$

for every $x \in \mathbb{R}$, respectively. Draw the graphs of $h_1, h_2, h_3$ and find the solutions to the equations

$$h_1(x) = 0 \quad \& \quad h_2(x) = 0 \quad \& \quad h_3(x) = 0.$$

**[4 points]**
**(ii)** Does the rule $x \mapsto f(x)$, where

$$f(x) = \frac{1}{x^{2019} + 2020x^2 + 2020},$$

define a function from $\mathbb{R}$ to $\mathbb{R}$?
**[2 points]**

**Exercise 4.** **(i)** Let $f : \mathbb{R}^{+*} \to \mathbb{R}$ defined by

$$f(x) = 5x^{2020} + \frac{1}{2}\ln(x),$$

for every $x \in \mathbb{R}$. Find the second derivative $f''(x)$, where $x \in \mathbb{R}^{+*}$.
**[3 points]**
**(ii)** Let $g : \mathbb{R} \to \mathbb{R}$ defined by

$$g(x) = \ln\left(x^{2020} + \exp(x)\right),$$

for every $x \in \mathbb{R}$. Find the derivative $g'(x)$, where $x \in \mathbb{R}$.
**[3 points]**

**Exercise 5.** **(i)** Calculate the integral

$$\int_0^1 \left(5x^{2020} - \exp(x)\right) dx.$$

**[3 points]**
**(ii)** Calculate the integral

$$\int_0^1 \exp^2(x) dx.$$

**[3 points]**

**Exercise 6.** **(i)** Calculate the integral

$$\int_0^{\frac{\pi}{2}} \cos^2(x) dx.$$

**[3 points]**

**(ii)** Calculate the integral

$$\int_0^{\frac{\pi}{2}} \cos(t)\sqrt{\sin(t)} dt.$$

**[3 points]**

**Discussion.** Monday 03. February 2020, in the Lecture.