



# Mathematics for Natural Scientists I

## Sheet 14

**Exercise 1.** Let  $a, b \in \mathbb{R}$ , such that  $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$ . Calculate the following integral:

$$\int_a^b \tan(t) dt.$$

[4 points]

**Exercise 2.** Let  $a, b \in \mathbb{R}$ , such that  $-1, 1 \notin [a, b]$ . Calculate the following integral:

$$\int_a^b \frac{1}{1-x^2} dx.$$

[Hint: Use the equality

$$\frac{1}{1-x^2} = \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x}.]$$

[4 points]

**Exercise 3.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function.

(i) Show the following equality:

$$\int_2^4 t f(t^2) dt = \frac{1}{2} \int_4^{16} f(x) dx.$$

[Hint: Use the substitution rule.]

[2 points]

(ii) Calculate the following integral:

$$\int_2^4 t^2 f(t^3) dt.$$

[2 points]

**Exercise 4.** Let  $t \neq 0$ . Calculate the integral

$$\int e^{tx} \sin(x) dx.$$

[4 points]

**Submission.** Monday 03. February 2020, in the Lecture.

**Discussion.** Monday 03. February 2020, in the Lecture.