LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Winter term 19/20
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Mathematics for Natural Scientists I

Sheet 13

Exercise 1. Let $\phi \in \mathcal{T}[a,b]$ be a step-function.
(i) Show that there are $m, M \in \mathbb{R}$ such that

$$m \leq \phi \leq M.$$

[2 points]
(ii) Show that

$$\overline{\int_a^b} \phi(x)dx = \underline{\int_a^b} \phi(x)dx = \int_a^b \phi(x)dx.$$

[2 points]
[**Hint**. Show that $\int_a^b \phi(x)dx$ is the greatest lower bound of $A(\phi)$ and the least upper bound of $B(\phi)$.]

Exercise 2. Let the Dirichlet-Function $\mathtt{Dir} : [0,1] \to \mathbb{R}$, defined by

$$\mathtt{Dir}(x) := \begin{cases} 1 & , x \in \mathbb{Q} \cap [0,1] \\ 0 & , x \in \mathbb{I} \cap [0,1], \end{cases}$$

Show that

$$\overline{\int_0^1} \mathtt{Dir}(x)dx = 1 \quad \& \quad \underline{\int_0^1} \mathtt{Dir}(x)dx = 0.$$

[4 points]

Exercise 3. Calculate the following integrals:

$$\int_0^1 \left(x^{2020} + 3x^{2019}\right) dx,$$

$$\int_\pi^{2\pi} \sin x dx,$$

$$\int_0^{\frac{\pi}{2}} \cos x dx,$$

$$\int_0^1 \exp(x) dx.$$

[4 points]

Exercise 4. Let the function $f : \mathbb{R}^{+*} \to \mathbb{R}$ be defined by

$$f(x) = x \ln(x) - x,$$

for every $x \in \mathbb{R}$.

(i) Find the derivative $f'(x_0)$, where $x_0 \in \mathbb{R}^{+*}$.

[2 points]

(ii) Calculate the integral

$$\int_1^2 \ln(x) dx.$$

[2 points]

Submission. Wednesday 29. January 2020, in the Exercise-session.

Discussion. Wednesday 29. January 2020, in the Exercise-session.