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Winter term 19/20
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Mathematics for Natural Scientists I

Sheet 11

Exercise 1. Using the Definition 2.4.1. show that for every $x_0 \in \mathbb{R}$

$$\exp'(x_0) = \exp(x_0).$$

[4 points]

[**Hint**: Use the equality

$$\lim_{x \longrightarrow 0} \frac{\exp(x) - 1}{x} = 1.]$$

Exercise 2. Let $f : D \to \mathbb{R}$ be differentiable in D. If the derivative function $f' : D \to \mathbb{R}$ is at $x_0 \in D$ differentiable, then we call the derivative

$$\frac{d^2 f}{dx^2}(x_0) = f''(x_0) = \big(f'\big)'(x_0)$$

the *second derivative* of f at x_0.

Let $f : \mathbb{R} \to \mathbb{R}$, defined by

$$f(x) = 7x^2 + 6x^3,$$

for every $x \in \mathbb{R}$. Show that

$$f''(x_0) = 14 + 36x_0,$$

for every $x_0 \in \mathbb{R}$.

[4 points]

Exercise 3. A function $f : \mathbb{R} \to \mathbb{R}$ is called *even*, if

$$f(-x) = f(x),$$

for every $x \in \mathbb{R}$, and it is called *odd*, if

$$f(-x) = -f(x),$$

for every $x \in \mathbb{R}$.
Let $p : \mathbb{R} \to \mathbb{R}$ be the polynomial function

$$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n, \qquad a_0, a_1, a_2, \ldots a_n \in \mathbb{R}.$$

(i) Show that p is an even function if and only $a_k = 0$, for every odd number $k \in \{0, \ldots, n\}$.
[2 points]
(ii) If p is an even function, show that its derivative p' is an odd function.
[2 points]

Exercise 4. Let $f, g : D \to \mathbb{R}$, $x_0 \in D$ and x_0 is an accumulation point of $D \setminus \{x_0\}$. Suppose also that f is differentiable at x_0, g is continuous at x_0, and $f(x_0) = 0$. Show that the function $fg : D \to \mathbb{R}$, where $(fg)(x) = f(x)g(x)$, for every $x \in D$, is differentiable at x_0 with

$$(fg)'(x_0) = f'(x_0)g(x_0).$$

[4 points]

Submission. Wednesday 08. January 2020, in the Exercise-session.

Discussion. Wednesday 08. January 2020, in the Exercise-session.