Dr. Iosif Petrakis

Winter term 19/20
16.12.2019

Mathematics for Natural Scientists I

Sheet 10

Exercise 1. Let the function $\sqrt{\ }: \mathbb{R}^+ \to \mathbb{R}^+$, defined by $x \mapsto \sqrt{x}$, for every $x \in \mathbb{R}^+$. Show that the function $\sqrt{\ }$ is continuous on $\mathbb{R}^+$.
[4 points]

Exercise 2. **(i)** Let $D \subseteq \mathbb{R}$, $x_0 \in D$, and $f : D \to \mathbb{R}$ a real function on D. Show that

$$\lim_{x \longrightarrow x_0} f(x) = f(x_0) \Leftrightarrow \lim_{h \longrightarrow 0} f(x_0 + h) = f(x_0).$$

[2 points]
(ii) If $x, y \in \mathbb{R}^{+*}$, show that

$$\ln(x \cdot y) = \ln(x) + \ln(y),$$

[2 points]
[**Hint:** use the equality $\exp(x + y) = \exp(x)\exp(y)$.]

Exercise 3. **(i)** Let the function $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = x^3$, for all $x \in \mathbb{R}$. With the use of the Definition 2.4.1. show that

$$f'(x_0) = 3x_0^2,$$

for all $x_0 \in \mathbb{R}$.
[2 points]
(ii) Let the function $g : \mathbb{R} \to \mathbb{R}$, where $g(x) = x^4$, for all $x \in \mathbb{R}$. Witht the use of the Definition 2.4.1. show that

$$g'(x_0) = 4x_0^3,$$

for all $x_0 \in \mathbb{R}$.
[2 points]

Exercise 4. Let $n \in \mathbb{N}$ with $n > 0$ and let the function $f : \mathbb{R} \to \mathbb{R}$, where

$$f(x) = x^n,$$

for all $x \in \mathbb{R}$. With the use of the Definition 2.4.1. show that

$$f'(x_0) = nx_0^{n-1},$$

for all $x_0 \in \mathbb{R}$.
[4 points]
[**Hint**: Use the equality

$$(x_0 + h)^n = \sum_{k=0}^{n} \binom{n}{k} x_0^k \cdot h^{n-k}.]$$

Submission. Wednesday 08. January 2020, in the Exercise-session.

Discussion. Wednesday 08. January 2020, in the Exercise-session.