



Mathematics for Natural Scientists I

Sheet 1

Exercise 1. Let $X = \{5, 10, 15, 20\}$. Find all subsets of X .

[4 points]

Exercise 2. If X is a set and A, B, C are subsets of X , show the following:

(i) $(A \cap B) \cup A = A$.

[0.5 point]

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

[1 point]

(iii) $A \cap A' = \emptyset$ and $A \cup A' = X$.

[0.5 point]

(iv) $(A')' = A$.

[0.5 point]

(v) $(A \cap B)' = A' \cup B'$.

[1.5 points]

Exercise 3. (i) Is the function $g : \mathbb{Z} \rightarrow \mathbb{Z}$, defined by $z \mapsto z^2$, a surjection?

[0.5 point]

(ii) Give an example of a function $h : \mathbb{N} \rightarrow \mathbb{N}$, which is neither injective nor surjective.

[1 point]

(iii) Find functions $f : \mathbb{N} \rightarrow \mathbb{N}$ and $g : \mathbb{N} \rightarrow \mathbb{N}$ such that $f \circ g \neq g \circ f$.

[1 point]

(iv) Find functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ and $g : \mathbb{Z} \rightarrow \mathbb{Z}$ such that $f \circ g = g \circ f$.

[1.5 point]

Exercise 4. Let X, Y, Z, W be sets, $f : X \rightarrow Y$, $g : Y \rightarrow Z$ and $h : Z \rightarrow W$. Show the following:

(i) $\text{id}_Y \circ f = f$.

[0.5 point]

(ii) $h \circ (g \circ f) = (h \circ g) \circ f$.

[1.5 points]

(iii) If f, g are injective, then $g \circ f$ is injective.

[1 point]

(iv) If f, g are surjective, then $g \circ f$ is surjective.

[1 point]

Exercise 5. Let $f : X \rightarrow Y$. Show that f is a bijection if and only if there is $g : Y \rightarrow X$ such that $f \circ g = \text{id}_Y$ and $g \circ f = \text{id}_X$.

[4 points]

[Hint: You need to show that if f is a bijection there is such a function $g : Y \rightarrow X$, and you also need to show that if there is such a function $g : Y \rightarrow X$, then f is a bijection.]

Submission. Wednesday 23. October 2019, in the Exercise-session.

Discussion. Wednesday 23. October 2019, in the Exercise-session.