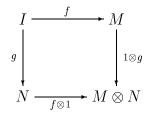
Problem set for Quantum Groups and Noncommutative Geometry

- (33) (Linear Algebra) For $U \subseteq V$ define $U^{\perp} := \{ f \in V^* | f(U) = 0 \}$. For $Z \subseteq V^*$ define $Z^{\perp} := \{ v \in V | Z(v) = 0 \}$. Show that the following hold:
 - (a) $U \subseteq V \Longrightarrow U = U^{\perp \perp}$;
 - (b) $Z \subseteq V^*$ and dim $Z < \infty \Longrightarrow Z = Z^{\perp \perp}$;
 - (c) $\{U \subseteq V | \dim V/U < \infty\} \cong \{Z \subseteq V^* | \dim Z < \infty\}$ under the maps $U \mapsto U^{\perp}$ and $Z \mapsto Z^{\perp}$.
- (34) Let $V = \bigoplus_{i=1}^{\infty} \mathbb{K} x_i$ be an infinite-dimensional vector space. Find an element $g \in (V \otimes V)^*$ that is not in $V^* \otimes V^*$ $(\subseteq (V \otimes V)^*)$.
- (35) For morphisms $f: I \to M$ and $g: I \to N$ in a monoidal category we define $(f \otimes 1: N \to M \otimes N) := (f \otimes 1_I)\rho(I)^{-1}$ and $(1 \otimes g: M \to M \otimes N) := (1 \otimes g)\lambda(I)^{-1}$. Show that the diagram



commutes.

(36) Let G be a finite group and $\mathbb{K}^G := \mathbb{K}[G]^*$ the dual of the group algebra. Show that \mathbb{K}^G is a Hopf algebra and that each module structure $\mathbb{K}[G] \otimes M \to M$ translates to the structure of a comodule $M \to \mathbb{K}^G \otimes M$ and conversely. Show that this defines a monoidal equivalence of categories. Describe the group valued functor \mathbb{K} - $c\mathcal{A}lg(\mathbb{K}^G, -)$ in terms of sets and their group structure.

Due date: Tuesday, 25.06.2002, 16:15 in Lecture Hall E41