Mathematisches Institut
der Universität München
Prof. Dr. B. Pareigis

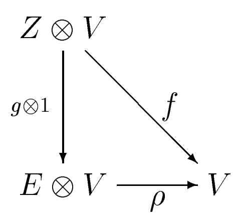
SS 2002
Set 8

Problem set for
Quantum Groups and Noncommutative Geometry

(29) Determine the structure of a covector space on a vector space V from the fact that $\mathrm{Hom}(V, W)$ is a vector space for all vector spaces W.

(30) The real unit circle $\mathcal{S}^1(\mathbb{R})$ carries the structure of a group by the addition of angles. Is it possible to make $\mathcal{S}^1$ with the affine algebra $\mathbb{K}[c, s]/(s^2 + c^2 - 1)$ into an affine algebraic group? (Hint: How can you add two points (x_1, y_1) and (x_2, y_2) on the unit circle, such that you get the addition of the associated angles?)
Find a group structure on the torus $\mathcal{T}$.

(31) Let V be a vector space. Show that there is a universal vector space E and homomorphism $\rho : E \otimes V \longrightarrow V$ (such that for each vector space Z and each homomorphism $f : Z \otimes V \longrightarrow V$ there is a unique homomorphism $g : Z \longrightarrow E$ such that



commutes). We call E and $\rho : E \otimes V \longrightarrow V$ a *vector space acting universally on* V.

(32) Let E and $\rho : E \otimes V \longrightarrow V$ be a vector space acting universally on V. Show that E has a uniquely determined structure of an algebra such that V becomes a left E-module.

Due date: Tuesday, 18.06.2002, 16:15 in Lecture Hall E41