

**Problem set for  
Quantum Groups and Noncommutative Geometry**

- (5) Let  $\mathcal{X}$  denote the plane curve  $y = x^2$ . Then  $\mathcal{X}$  is isomorphic to the affine line.
- (6) \* Let  $\mathbb{K}$  be an algebraically closed field. Let  $p$  be an irreducible square polynomial in  $\mathbb{K}[x, y]$ . Let  $\mathcal{Z}$  be the conic section defined by  $p$  with the affine algebra  $\mathbb{K}[x, y]/(p)$ . Show that  $\mathcal{Z}$  is naturally isomorphic either to  $\mathcal{X}$  or to  $\mathcal{U}$  from problems (3) resp. (5).
- (7) Let  $\mathcal{X}$  be an affine scheme with affine algebra

$$A = \mathbb{K}[x_1, \dots, x_n]/(p_1, \dots, p_m).$$

Define “coordinate functions”  $q_i : \mathcal{X}(B) \rightarrow B$  which describe the coordinates of  $B$ -points and identify these coordinate functions with elements of  $A$ .

- (8) Let  $S_3$  be the symmetric group and  $A := \mathbb{K}[S_3]$  be the group algebra on  $S_3$ . Describe the points of  $\mathcal{X}(B) = \mathbb{K}\text{-Alg}(A, B)$  as a subspace of  $\mathbb{A}^2(B)$ . What is the commutative part  $\mathcal{X}_c(B)$  of  $\mathcal{X}$  and what is the affine algebra of  $\mathcal{X}_c$ ?

Due date: Tuesday, 30.04.2002, 16:15 in Lecture Hall E41