Mathematisches Institut
der Universität München
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SS 2002
Set 1

Problem set for Quantum Groups and Noncommutative Geometry

(1) Determine the affine algebra of the functor "unit sphere" S^{n-1} in $\mathbb{A}^n$.

(2) Determine the affine algebra of the functor "torus" $\mathcal{T}$ and find an "embedding" of $\mathcal{T}$ into $\mathbb{A}^3$.

(3) Let $\mathcal{U}$ denote the plane curve $xy = 1$. Then $\mathcal{U}$ is not isomorphic to the affine line. (Hint: An isomorphism $\mathbb{K}[x, x^{-1}] \longrightarrow \mathbb{K}[y]$ sends x to a polynomial $p(y)$ which must be invertible. Consider the highest coefficient of $p(y)$ and show that $p(y) \in \mathbb{K}$. But that means that the map cannot be bijective.)
$\mathcal{U}$ is also called the *unit functor*. Can you explain, why?

(4) Let $\mathbb{K} = \mathbb{C}$ be the field of complex numbers. Show that the unit functor $\mathcal{U} : \mathbb{K}\text{-}c\mathcal{A}lg \longrightarrow \mathcal{S}et$ in Problem (3) is naturally isomorphic to the unit circle S^1. (Hint: There is an algebra isomorphism between the representing algebras $\mathbb{K}[e, e^{-1}]$ and $\mathbb{K}[c, s]/(c^2 + s^2 - 1)$.)

Due date: Tuesday, 23.04.2002, 16:15 in Lecture Hall E41