Mathematisches Institut
der Universität München
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WS 2001/02
Set 7

Problem set for
Advanced Algebra

(25) Let $(A \times B, p_A, p_B)$ be the product of A and B in $\mathcal{C}$. Then there is a natural isomorphism

$$\mathrm{Mor}(\text{-}, A \times B) \cong \mathrm{Mor}_{\mathcal{C}}(\text{-}, A) \times \mathrm{Mor}_{\mathcal{C}}(\text{-}, B).$$

(26) Let $\mathcal{C}$ be a category with finite products. Show that there is a bifunctor $\text{-} \times \text{-} : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$ such that $(\text{-} \times \text{-})(A, B)$ is the object of a product of A and B. We denote elements in the image of this functor by $A \times B := (\text{-} \times \text{-})(A, B)$ and similarly $f \times g$.

(27) Let $\mathcal{F} : \mathcal{C} \longrightarrow \mathcal{D}$ be an equivalence with respect to $\mathcal{G} : \mathcal{D} \longrightarrow \mathcal{C}$, $\varphi : \mathcal{GF} \cong \mathrm{Id}_{\mathcal{C}}$, and $\psi : \mathcal{FG} \cong \mathrm{Id}_{\mathcal{D}}$. Show that $\mathcal{G} : \mathcal{D} \longrightarrow \mathcal{C}$ is an equivalence. Show that $\mathcal{G}$ is uniquely determined by $\mathcal{F}$ up to a natural isomorphism.

(28) (a) Given $V \in \mathbb{K}\text{- Mod}$. For $A \in \mathbb{K}\text{- Alg}$ define

$$F(A) := \{f : V \longrightarrow A | f \ \mathbb{K}\text{-linear}, \forall v, w \in V : f(v) \cdot f(w) = 0\}.$$

Show that this defines a functor $F : \mathbb{K}\text{- Alg} \longrightarrow \mathrm{Set}$.

(b) Show that F has the algebra $D(V)$ as constructed in Exercise 2.1 (3) as a representing object.

Due date: Tuesday, 4.12.2001, 16:15 in Lecture Hall 138