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Functional Analysis II
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Excercise Sheet 2 for 3. 11. 2017

Let H be a separable Hilbert space.

2.1. Let $A : D(A) \to H$ and $B : D(B) \to H$ be two symmetric operators. Prove that if $A \subset B$ (i.e. B is an extension of A), then $B^* \subset A^*$. Deduce that if A is self-adjoint, then it has no symmetric extension except itself.

2.2. Prove that if $A : D(A) \to H$ is self-adjoint, then it is closed, i.e. its graph $\{(x, Ax) \in H \oplus H : x \in D(A)\}$ is a closed set in $H \oplus H$.

2.3. Assume that $A : D(A) \to H$ is a (densely defined) closed operator. Prove that $B = A^*A$ with $D(B) = \{x \in D(A) : Ax \in D(A^*)\}$ is a self-adjoint operator. Prove that the quadratic form domain $Q(B)$ is exactly $D(A)$.

2.4. Let $H = L^2(0,1)$ and $A = -\Delta$ with $D(A) = \{f \in C^2(0,1), f(0) = f(1) = 0\}$. Prove that A is a symmetric operator, but A is not a self-adjoint operator. Let $g(x) = x(1-x) \in D(A)$. Explain where is wrong in the following equalities

$$\text{“}4 = \langle Ag, Ag\rangle = \langle g, A^2 g\rangle = 0.\text{”}$$