Mathematisches Institut der LMU
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Functional Analysis II
Winter Semester 2017/18
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Excercise Sheet 1 for 27. 10. 2017

1.1. Assume that $x_n \rightharpoonup x$ weakly in a Hilbert space. Prove that

(i) $||x|| \le \liminf ||x_n||$.

(ii) If $||x_n|| \to ||x||$ then $x_n \to x$ strongly.

(iii) If $y_n \to y$ strongly then $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

1.2. Let $\{u_n\}$ be an orthonormal family in a separable Hilbert space and let A be a compact operator. Prove that $Au_n \to 0$ strongly.

1.3. Let $A : D(A) \to H$ be a (densely defined) unbounded operator. Prove that the following statements are equivalent:

(i) A is a symmetric operator, i.e. $\langle u, Av \rangle = \langle Au, v \rangle$ for all $u, v \in D(A)$.

(ii) $\langle u, Au \rangle \in \mathbb{R}$ for all $u \in D(A)$.

(iii) The adjoint operator A^* is an extension of A.

1.4. Let A be a compact operator (not necessarily symmetric) on a Hilbert space. Prove that there are orthonnormal bases $\{u_n\}$ and $\{v_n\}$, and a sequence of real numbers $\{\lambda_n\}$ with $\lim_{n\to\infty} \lambda_n = 0$ such that

$$Au_n = \lambda_n v_n, \ \forall n \in \mathbb{N}.$$

Hint: You can use the spectral theorem for A^*A.