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Mathematical Quantum Mechanics
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Tutorial Sheet 3

3.1. Let $\{u_n\}_{n=1}^{\infty}$ and $\{v_n\}_{n=1}^{\infty}$ be orthonormal families in a Hilbert space. Let $\{\lambda_n\}_{n=1}^{\infty}$ be a sequence of complex numbers. Consider the operator

$$A = \sum_{n=1}^{\infty} \lambda_n |v_n\rangle\langle u_n|.$$

Prove that if A is a compact operator then $\lambda_n \to 0$ as $n \to \infty$.

3.2. Let A be a linear operator in the Hilbert space. Prove that the following statements are equivalent

i) A is a bounded operator;
ii) $x_n \rightharpoonup x$ weakly implies $Ax_n \rightharpoonup Ax$ weakly.

3.3. Let $\{e_k\}$ be an orthonormal basis in a Hilbert space. Prove that the following statements are equivalent

i) $\phi_n \rightharpoonup \phi$ weakly
ii) $\{\phi_n\}$ is bounded and $\langle \phi_n, e_k\rangle \to \langle \phi, e_k\rangle$ for all k.

3.4. Let P ($P \neq 0, P \neq \mathrm{Id}$) be a projection operator, i.e., $P = P^2 = P^*$ and P is bounded. Prove that $\sigma(P) = \{0; 1\}$.