

Tutorial Sheet 2

2.1. Prove that if $\|f\|_{L^p} = 0$, $1 \leq p < \infty$ then $f = 0$ almost everywhere.

2.2. Prove that $f_n \rightharpoonup f$ weakly and $\|f_n\| \rightarrow \|f\|$ then $f_n \rightarrow f$ strongly.

2.3. Let $f \in L^1(\Omega)$.

- (i) Prove that $\hat{f} \in L^\infty(\Omega) \cap C(\Omega)$
- (ii) Prove that \hat{f} is uniformly continuous.

2.4. Let $f \in L^p(\Omega)$ and define $f_h(x) = f(x+h)$, $h \in \Omega$. Prove that

$$\lim_{|h| \rightarrow 0} \|f_h - f\|_{L^p} = 0.$$

2.5. Let $f(x) = e^{-\pi x^2}$ in \mathbb{R} . Prove that $\hat{f}(k) = e^{-\pi k^2}$.