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# Tutorial Sheet 10

**10.1.** Consider the free Schrödinger dynamics

$$u(t) = e^{it\Delta} u_0, \quad u_0 \in L^1 \cap L^2(\mathbb{R}^d).$$

Prove that $\lim_{|t|\to\infty} \|u(t)\|_{L^p} = 0$ for any $2 < p \leq \infty$.

**10.2.** Consider the operator $A = -\Delta - |g\rangle\langle g|$ where $g \in C_c^\infty(\mathbb{R}^d)$ with $d > 2$. Prove that $\lim_{t\to\infty} e^{itA} e^{it\Delta}$ exists strong in $L^2(\mathbb{R}^3)$.

**10.3.** Let $A$, $B$ and $C$ be self-adjoint operators on a separable Hilbert space $\mathcal{H}$. Prove that if $\lim_{t\to\infty} e^{itA} e^{-itB}$ and $\lim_{t\to\infty} e^{itB} e^{-itC}$ exist strongly in $\mathcal{H}$ then so does $\lim_{t\to\infty} e^{itA} e^{-itC}$.

**10.4.** Let $A : D(A) \to \mathcal{H}$ be a self-adjoint operator and $\{B_n\}$ be a sequence of bounded self-adjoint operators such that $\lim_{n\to\infty} \|B_n\| = 0$. Prove that

$$\lim_{n\to\infty} \|e^{it(A+B_n)} - e^{itA}\| = 0, \quad \forall t > 0.$$