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Mathematical Quantum Mechanics
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Tutorial Sheet 1

1.1. Assume that $f_n \to f$ and $g_n \to g$ almost everywhere. Prove that $f_n + g_n \to f + g$ almost everywhere.

1.2. Let A be a (unbounded) densely defined operator in a Hilbert space.

(i) Prove that A^* is unique.
(ii) Prove that A is symmetric if and only if $A \subset A^*$.

1.3. Let A and B be (unbounded) densely defined operators in a Hilbert space.

(i) Prove that if $A \subset B$ then $B^* \subset A^*$.
(ii) Deduce that if A is self-adjoint then it has no extension except itself.

1.4. Consider the operator $A = i\dfrac{\mathrm{d}}{\mathrm{d}t}$ with

$$D(A) = \{f \in L^2(0,1) : f' \text{ is continuous and } f(0) = f(1) = 0\}.$$

Prove that A is a symmetric operator and $A \subsetneq A^*$.

1.5. Let (Ω, Σ, μ) be a measurable space. Let $A_n \in \Sigma$, $n = 1, 2, \dots$ such that $A_1 \subset A_2 \subset \dots \subset A_n \subset \dots$ and let $A = \cup_{n=1}^{\infty} A_n$. Prove that

$$\mu(A) = \lim_{n\to\infty} \mu(A_n).$$