

Tutorial 8 for 07/12/2018

Ex 1: Let $A: D(A) \rightarrow H$ be a symmetric operator.

Prove that i) $A = \overline{A}$ if A is bounded

ii) $A \subset \overline{A}$ if A is unbounded

iii) $\overline{A^*} = A^*$.

Ex 2: Prove the ellipticity

$$\|u\|_{H^2(\mathbb{R}^d)} \leq C(\|u\|_{L^2} + \|\Delta u\|_{L^2(\mathbb{R}^d)}) \quad \forall u \in H^2(\mathbb{R}^d)$$

where C is a constant independent of u .

Ex 3: Let $A: D(A) \rightarrow H$ be a closed operator.

Prove that A^{-1} is closed if A is injective.

Ex 4: Let $f \in H^1(0,1)$. Prove that

$$f(0) \leq \frac{2}{\sqrt{3}} \|f\|_{H^1}, \quad f(1) \leq \frac{2}{\sqrt{3}} \|f\|_{H^1}$$