

## Tutorial 7 for 30/11/2018

Ex 1: Let  $A: D(A) \rightarrow H$  and  $B: D(B) \rightarrow H$  be two (densely defined) unbounded operators. Prove that if  $A \subset B$  then  $B^* \subset A^*$ .

Ex 2: Let  $A = -i \frac{d}{dx}$  be an operator on  $H = L^2(\mathbb{R})$

Prove that i)  $A$  is unbounded but not self-adjoint on  $C_c^\infty(\mathbb{R})$ .

ii)  $A$  is self-adjoint on  $H^1(\mathbb{R})$

Ex 3: Let  $A: D(A) \rightarrow H$  be an unbounded self-adjoint operator. Use the Spectral theorem (multiplication operator version) to prove the Monotone Convergence theorem.

If  $\{f_n\} \subset B(\sigma(A))$  and  $|f_n(t)| \leq |f_{n+1}(t)|$  and  $f_n(t) \rightarrow f(t)$  pointwise, then

$$\|f_n(A)u - f(A)u\| \rightarrow 0 \quad \forall u \in H.$$