Tutorial 6 for 23/11/18

Ex 1: Let A and B be bounded linear operator on a separable Hilbert space H. Verify that

$$(A+B)^* = A^* + B^*, \quad (\alpha A)^* = \bar{\alpha} A^* \ (\alpha \in \mathbb{C}).$$

$$A^{**} = A, \quad (AB)^* = B^* A^*, \quad \|A^*\| = \|A\|,$$

$$\|A^*A\| = \|A\|^2 = \|AA^*\|.$$

Ex 2: Let $\{e_n\}$ be an ONB of H and T be a compact operator on H. Show that $\|Te_n\| \to 0$ as $n \to \infty$

Ex 3: Prove that if T is bdd linear s.a. operator on H and $\sigma(T) = \{0\}$ then $T \equiv 0$.

Ex 4: Let A be a ~~bdd~~ linear s.a. operator on H and A is positive. Show that $\sigma(A) \subset [0, +\infty)$.

Ex 5: Let $\{e_n\}$ be ONB of separable H. Consider sequence of operator $T_L = L|e_L\rangle\langle e_L|$. Show that

i) $\sigma(T_L) = \{0, L\}$

ii) $T_L \not\to 0$

Ex 6: Let $\{e_k\}$ be an ONB of H. Show that

$$\{\varphi_n \to \varphi \text{ weakly}\} \text{ iff } \{\varphi_n \text{ is bdd and } \langle \varphi_n, e_k \rangle \to \langle \varphi, e_k \rangle \ \forall k\}$$