Tutorial 4 for 09/11/2018

<u>Ex1</u>: let A be a linear operator on a separable Hilbert space $\mathcal{H}$. Prove that

$$A \text{ is bounded} \Leftrightarrow \{ x_n \to x \Rightarrow Ax_n \to Ax \}$$

<u>Ex2</u>: Let $f \in H^1(\mathbb{R}^d)$. Show that

$$\int_{\mathbb{R}^d} |\nabla f|^2 = \lim_{h \to 0} \int \left| \frac{f(x+h) - f(x)}{h} \right|^2 dx$$

<u>Ex3</u>: Show that $\Delta \left(\frac{1}{|x|^{d-2}} \right) = 0$ on $\mathbb{R}^d \setminus \{0\}$, $\forall d \geq 3$

<u>Ex4</u>: Prove that the following statements are equivalent

i) $f \in H^2(\mathbb{R}^d)$, $g \in L^2(\mathbb{R}^d)$ and $-\Delta f + f = g$

ii) $f \in H^1(\mathbb{R}^d)$, $g \in L^2(\mathbb{R}^d)$ and $\langle \nabla f, \nabla \varphi \rangle + \langle f, \varphi \rangle = \langle g, \varphi \rangle \quad \forall \varphi \in H^1(\mathbb{R}^d)$

<u>Ex5</u>: Prove that if $f_n \to f$ and $\nabla f_n \to g$ strongly in $L^2(\mathbb{R}^d)$ then $f \in H^1(\mathbb{R}^d)$ and $\nabla f = g$.
Here f_n be a sequence in $H^1(\mathbb{R}^d)$.