Tutorial 3 for 02/11/2018

Ex1: Let $f_n \rightharpoonup f$ weakly in $L^2(\mathbb{R}^d)$. Prove that if $\|f_n\|_{L^2} \to \|f\|_{L^2}$ then $f_n \to f$ strongly in $L^2(\mathbb{R}^d)$

Ex2: Let $f \in L^1(\mathbb{R}^d)$. Show that $\hat{f}(k) \to 0$ as $|k| \to \infty$

Ex3: Let $T \in D'(\mathbb{R}^2)$. Show that

$$\partial_{x_1} \partial_{x_2} T = \partial_{x_2} \partial_{x_1} T$$

Ex4: Let $f \in C_c^\infty(\mathbb{R}^d)$ and $T_g \in D'(\mathbb{R}^d)$. Show that $gf \in D'$ and $\frac{\partial}{\partial x_1}(gf) = \left(\frac{\partial}{\partial x_1} g\right) f + g\left(\frac{\partial f}{\partial x_1}\right)$.

Ex5: Let $V(x) = -\frac{1}{|x-x_1|} - \frac{1}{|x-x_2|}$. Prove that there exists constant C such that

$$\int_{\mathbb{R}^3} |\nabla u|^2 + \int V(x)|u(x)|^2 \, dx \geq -C$$

where $u \in H^1(\mathbb{R}^3)$ and $\|u\|_{L^2} = 1$.