Mathematisches Institut der LMU
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Mathematical Quantum Mechanics
Winter Semester 2018-2019
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Mathematical Quantum Mechanics

Midterm Exam

Nachname: ____________ Vorname: ____________

Geburtstag: ____________ Matrikelnr.: ____________

Studiengang: ____________ Fachsemester: ____________

Please place your identity and student ID cards on the table so that they are clearly visible. Switch off your mobile phone and all other electronic devices.

Please write your name and your solutions on the provided sheets. You can use your notes. You can refer to all results discussed in the lectures and homework sheets. You can try any problem and collect partial credits.

You have 90 minutes. We count 100 points as the perfect score. Good luck!

Problems	1	2	3	4	$\sum$
Maximum points	15	20	35	40	110
Scored points					

Problem 1. (15 points) Determine whenever the following mapping is a distribution in $\mathscr{D}'(\mathbb{R})$? Justify your claim.

$$T(\varphi) = \sum_{n=1}^{\infty} n^{-2}\varphi(n^{-1}), \quad \forall \varphi \in C_c^\infty(\mathbb{R}).$$

Problem 2. (20 points) Let A be a bounded self-adjoint operator on a Hilbert space. Assume that $Au = \lambda u$, with λ an eigenvalue and $u \neq 0$ an eigenvector. Prove that

$$f(A)u = f(\lambda)u, \quad \forall f \in C(\mathbb{R}).$$

Problem 3. Let $G(x) = e^{-\pi|x|^2}$ and define the operator $A : D(A) \to L^2(\mathbb{R}^3)$

$$Au = -\Delta u - G * u, \quad \forall u \in D(A) = H^2(\mathbb{R}^3).$$

(i) (15 points) Prove that A is a self-adjoint operator.

(ii) (20 points) Let $u(t)$ be the solution to the equation

$$i\frac{d}{dt}u(t) = Au(t) \quad \forall t \in \mathbb{R}, \quad u(0) = u_0 \in D(A).$$

Prove that there exists a constant $C > 0$ independent of t and u_0 such that

$$\|u(t)\|_{H^2(\mathbb{R}^3)} \leq C\|u(0)\|_{H^2(\mathbb{R}^3)}, \quad \forall t \in \mathbb{R}.$$

Hint: You can show that $\|Au(t)\|_{L^2}$ is independent of t.

Problem 4. Define the operator $A : D(A) \to L^2(\mathbb{R})$ by

$$(Au)(x) = -u''(x) + |x|u(x), \quad \forall u \in D(A) = C_c^\infty(\mathbb{R}).$$

(i) (20 points) Prove that $A \geq c_0$ for a constant $c_0 > 0$.

(ii) (20 points) Let A_F be the Friedrichs' self-adjoint extension of A. Prove that A_F^{-1} is a compact operator.

Hint: You may show that $A_F^{-1/2}$ is compact.