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Mathematical Quantum Mechanics
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Homework Sheet 15 for 8. 2. 2019

15.1. Let A be a mixed state on a Hilbert space H (i.e. a positive trace class operator with $\mathrm{Tr}A = 1$). Prove that for any nonnegative convex function $f : \mathbb{R} \to \mathbb{R}$ and for any orthonormal basis $\{u_n\}_n$ for H we have

$$\mathrm{Tr}[f(A)] \geq \sum_n f(\langle u_n, Au_n \rangle).$$

Hint: You can use the spectral decomposition of A.

15.2. Let ρ_1 be a mixed state on a Hilbert space H_1. Prove that for every Hilbert space H_2 with $\dim H_2 \geq \dim H_1$ (both can be $+\infty$), there exists a pure state $\rho = |\Psi\rangle\langle\Psi|$ with $\Psi \in H_1 \otimes H_2$ such that

$$\rho_1 = \mathrm{Tr}_{H_2}\rho \quad \text{(partial trace over the space } H_2\text{)}$$

i.e. $\mathrm{Tr}[\rho_1 A] = \mathrm{Tr}(\rho A \otimes 1_{H_2})$ for all compact operators A on H_1.

15.3. Let A and B be two self-adjoint operators on a Hilbert space H such that

$$A \geq B > 0.$$

i) Prove that $B^{-1} \geq A^{-1}$.

ii) Prove that $\sqrt{A} \geq \sqrt{B}$.
Hint: HW 9.1 may be helpful.

15.4. Let A and B be two positive self-adjoint operators on a Hilbert space H. Prove that

$$\frac{1}{A} + \frac{1}{B} \geq \frac{4}{A+B}.$$