Mathematisches Institut der LMU
Prof. P. T. Nam, Prof. A. Scrinzi
D.T. Nguyen

Mathematical Quantum Mechanics
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Homework Sheet 10 for 7. 1. 2019

10.1. Let A be a compact operator on a Hilbert space H. Let $\{B_n\}$ be a sequence of bounded self-adjoint operators on H such that $B_n \to 0$ strongly, i.e.

$$\|B_n x\| \to 0, \quad \forall x \in H.$$

Prove that $AB_n \to 0$ in operator norm.

10.2. Let $A : D(A) \to H$ and $B : D(B) \to H$ be two self-adjoint operators such that $(A+i)^{-1} - (B+i)^{-1}$ is a compact operator. Prove that

$$\sigma_{\text{ess}}(A) = \sigma_{\text{ess}}(B).$$

10.3. Let $G \in L^1(\mathbb{R}^3, \mathbb{R})$ and define $A : D(A) \to L^2(\mathbb{R}^3)$ by

$$A = -\Delta - (G * |x|^{-1}), \quad D(A) = H^2(\mathbb{R}^3).$$

Prove that A is a self-adjoint operator and $\sigma_{\text{ess}}(A) = [0, \infty)$.

10.4. Let $a > 0$ and define $A : D(A) \to L^2(\mathbb{R}^3)$ by

$$A = -\Delta + \frac{a}{|x|}, \quad D(A) = H^2(\mathbb{R}^3).$$

Prove that A is self-adjoint and $\sigma(A) = [0, \infty)$.

10.5. (Christmas bonus) Prove that the operator A in Problem 10.4 has no eigenvalues.