SPECTRAL DAYS 2012

TITLES AND ABSTRACTS*

M. Aizenman: *Resonant Delocalization for Schrödinger Operators with Random Potential on Tree Graphs*

We consider self adjoint operators of the form $H = T + V$ acting in the Hilbert space of square integrable functions on regular tree graphs, with T the graph incidence operator and V a random potential. Of particular interest is the existence and location of a 'mobility edge', which marks a transition between spectral regimes of pure point versus absolutely continuous spectra, where the unitary evolution generated by H exhibits different conductive and dynamical properties. It is shown that a mechanism of relevance is the formation of extended states through disorder enabled resonances, for which the exponential increase of the volume plays an essential role. By this mechanism extended states appear at weak disorder well beyond the spectrum of the operator's hopping term (T), including in a 'Lifshitz tail regime' of very low density of states. We also find that for bounded random potentials at weak disorder there is no mobility edge in the form that was envisioned before. Throughout the ac spectrum the evolution is ballistic. (Joint work with Simone Warzel.)

V. Bach: *Dynamical Renormalization Group*

In the past two decades a lot of effort has been invested into the spectral analysis of quantum field theoretic models of nonrelativistic matter which is coupled to the quantized radiation field. One of the approaches is the renormalization group based on the (smooth or sharp) Feshbach–Schur map. Ground state and resonance eigenvectors have been constructed by means of this method.

In a joint work with Jacob Schach Möller and Matthias Westrich it is shown that the long-time asymptotics of the dynamics of (some of) these systems can also be iteratively obtained by a similar method called the "Dynamical Renormalization Group". In particular it is shown that, for a given time scale $t \sim g^{-n}$ in powers of the coupling constant g, an effective Hamiltonian is derived, which generates the dynamics modula small errors. For $n = 2$, this effective Hamiltonian coincides with the well-known operator obtained in the van Hove (= weak coupling) limit

R. D. Benguria: *A New Estimate on the Two-Dimensional Indirect Coulomb Energy*

We prove a new lower bound on the indirect Coulomb energy in two dimensional quantum mechanics in terms of the single particle density of the system. The new universal lower bound is an alternative to the Lieb–Solovej–Yngvason bound with a smaller constant, $C = (4/3)^{3/2}\sqrt{5\pi - 1} \approx 5.90 < C_{LSY} = 192\sqrt{2\pi} \approx 481.27$, which also involves an additive gradient energy term of the single particle density. I will also review the analogous situation in 3-d. In 2-d this is joint work with P. Gallegos and M. Tusek. In 3-d is a joint collaboration with G. Bley and M. Loss.

* Updated Monday 9th April, 2012 at 17:49.

B. Davies: *Spectrum of a Feinberg–Zee Random Hopping Matrix*

This talk starts from a theorem of Chandler–Wilde, Chonchaiya and Lindner that the spectra of a certain class of infinite, random, tridiagonal matrices contain the unit disc almost surely. It obtains an analogous result for a more general class of random matrices whose spectra contain a hole around the origin. The presence of the hole forces substantial changes to the analysis.

M. Esteban: *Best Constants for Lieb–Thirring like Inequalities in Cylinders and their Relation to Symmetry Properties of Extremals for the Caffarelli–Kohn–Nirenberg Inequalities*

In this talk I will present recent work made in collaboration with J. Dolbeault and M. Loss about the relation between one-boundstate Lieb–Thirring inequalities on cylinders and the interpolation functional inequalities known as Cafffarelli–Kohn–Nirenberg inequalities. We show the relation between the best constants in both cases. We show that the total symmetry for the extremals for CKN inequalities implies the knowledge of the best constants for both inequalities. Finally, we give a large set of cases in which the symmetry can be proved.

S. Fournais: *Relativistic Scott Correction in Self-Generated Magnetic Fields*

We consider a large neutral molecule with total nuclear charge Z in a model with self-generated classical magnetic field and where the kinetic energy of the electrons is treated relativistically. To ensure stability, we assume that $Z\alpha < 2/\pi$, where α denotes the fine structure constant. We are interested in the ground state energy in the simultaneous limit $Z \to \infty$, $\alpha \to 0$ such that $\kappa = Z\alpha$ is fixed. The leading term in the energy asymptotics is independent of κ, it is given by the Thomas-Fermi energy of order $Z^{7/3}$ and it is unchanged by including the self-generated magnetic field. We prove the first correction term to this energy, the so-called Scott correction of the form $S(\alpha Z)Z^2$. This extends the result of Solovej–Sorensen–Spitzer on the Scott correction for relativistic molecules to include a self-generated magnetic field. Furthermore, we show that the corresponding Scott correction function S, is unchanged by including a magnetic field. We also prove new Lieb–Thirring inequalities for the relativistic kinetic energy with magnetic fields.
This is joint work with László Erdős and Jan Philip Solovej.

R. Frank: *Properties of Multi-Polaron Systems*

We review some recent results about multi-polaron systems. We prove that if the electronic Coulomb repulsion is large enough, there is no multi-polaron binding of any kind. We study this binding-unbinding transition in detail and we discuss symmetry properties of the ground state in the binding regime. As a second topic, we consider the transition from many-body collapse to the existence of a thermodynamic limit for large systems. While (most of) these results are also valid for quantized fields (the Fröhlich model), in the talk we will focus on the technically easier case of classical fields (the Pekar-Tomasevich model). The talk is based on joint works with R. D. Benguria, E. H. Lieb, R. Seiringer and L. E. Thomas.

J. Fröhlich: *Electrons and Light – QED light*

I will review a variety of mathematical results on the quantum theory of electrons and their interactions with the classical and quantized electro-magnetic field. In particular, I will describe results on atomic spectroscopy and on the ionization of atoms by laser pulses.

F. Germinet: *Anderson Localization for Delone Hamiltonians*

We prove that a large family of Delone Schrödinger operators exhibit Anderson localization. The proof relies on the analysis of random Schrödinger operators with a Delone background potential and a Delone-Bernoulli random potential. Joint work with P. Müller and C. Rojas-Molina.

B. Helffer: *A Magnetic Characterization of Minimal Partitions*

(after B. Helffer, T. Hoffmann-Ostenhof)

S. Jitomirskaya: *Quasiperiodic Schrödinger Operators and their Rational Frequency Approximants*

As, beginning with the famous Hofstadter's butterfly, all numerical studies of spectra of quasiperiodic operators are actually performed for their rational frequency approximants, the questions of continuity upon such approximation are of fundamental importance. The fact that continuity issues may be delicate is illustrated by the recently discovered discontinuity of the Lyapunov exponent for non-analytic potentials. We will report on two results: first, joint with C. Marx, showing that up to sets of zero Lebesgue measure, the absolutely continuous spectrum of analytic quasiperiodic Schrödinger operators can be obtained asymptotically from the intersections over the phase of the spectra of periodic frequency approximants, thus confirming a conjecture of Y. Last, and second, joint with R. Mavi, that measure of the spectrum is continuous upon such approximation even for rough potentials.

A. Joye: *Correlated Markov Quantum Walks*

We consider the discrete time unitary dynamics given by a quantum walk on the d-dimensional lattice performed by a quantum particle with internal degree of freedom, called coin state, according to the following iterated rule: a unitary update of the coin state takes place, followed by a shift on the lattice, conditioned on the coin state of the particle. We study the large time behavior of the quantum mechanical probability distribution of the position observable on the lattice when the sequence of unitary updates is given by a space-periodic function of a Markov chain in time. When averaged over the randomness, this distribution is shown to display a drift proportional to time whereas its centered counterpart displays a diffusive behavior. Moderate and large deviation principles are also proven to hold.
Joint work with Eman Hamza, Cairo.

A. Klein: *Local Behavior of Solutions of Schrödinger Equations and Bounds on the Density of States for Schrödinger Operators*

We study the local behavior of approximate solutions of stationary Schrödinger equations and establish a local decomposition into a homogeneous harmonic polynomial and a lower order term with explicit estimates. Combining this result with a version of Bourgain and Kenig's quantitative unique continuation principle we prove log-Hölder continuity of the density of states for Schrödinger operators in two and three dimensions. (Joint work with J. Bourgain.)

F. Klopp: *Level Statistics at Spectral Edges*

We will discuss recent results on level statitics at spectral edges for various random models.

M. Lewin: *Existence of the Thermodynamic Limit for Disordered Coulomb Quantum Systems*

In this talk I will explain how to prove the existence of the thermodynamic limit for a many-body quantum crystal in which the electrons are quantum and the nuclei are classical point particles, with disordered locations and charges around a lattice. This is a collaboration with Xavier Blanc, based on previous work with Christian Hainzl and Jan Philip Solovej.

O. Matte: *The Mass Shell in the Semi-Relativistic Pauli-Fierz Model*

In this talk we consider the semi-relativistic Pauli-Fierz model for a single free electron interacting with the quantized radiation field. Employing a variant of Pizzo's iterative analytic perturbation theory we construct a sequence of ground state eigenprojections of infra-red cutoff, dressing transformed fiber Hamiltonians and prove its convergence, as the cutoff goes to zero. Its limit is the ground state eigenprojection of a certain Hamiltonian unitarily equivalent to a renormalized fiber Hamiltonian acting in a coherent state representation space. The ground state energy is an exactly two-fold degenerate eigenvalue of the renormalized Hamiltonian, while it is not an eigenvalue of the original fiber Hamiltonian unless the total momentum is zero. These results hold true, for total momenta inside a ball about zero of arbitrary radius, provided that the coupling constant is sufficiently small (also depending on the ultra-violet cutoff). Along the way we prove twice continuous differentiability and strict convexity of the ground state energy as a function of the total momentum inside that ball.
The talk is based in joint work with Martin Könenberg (Vienna).

B. Schlein: *A Central Limit Theorem for Many Body Quantum Dynamics*

We study the time evolution of bosonic quantum systems in the mean field regime. We show that the fluctuations around the limiting Hartree dynamics satisfy a central limit theorem. Interestingly, the variance of the limiting Gaussian distribution can be expressed in terms of a time-dependent Bogoliubov transformation arising from the analysis of the evolution of initial coherent states.

R. Seiringer: *How much Energy does it Cost to Make a Hole in the Fermi Sea?*

Lieb–Thirring inequalities give a bound on power sums of the negative eigenvalues of a Schroedinger operator in terms of an L^p norm of the potential. We present an extension of these inequalities to analogous inequalities for perturbations of the continuous spectrum of the Laplacian by local potentials. In physics terms, we bound the change in energy of an ideal Fermi gas when the density is changed locally, which is an important quantity in condensed matter physics. Our bounds show that, up to a universal constant, the semiclassical approximation gives the correct answer. (This is joint work with R. Frank, M. Lewin and E. Lieb.)

S. Serfaty: *2D Classical Coulomb Gas and the Renormalized Energy*

In joint work with Etienne Sandier, we study the statistical mechanics of a two-dimensional classical Coulomb gas, particular cases of which also correspond to random matrix models. We connect the problem to the "renormalized energy" W, a Coulombian interaction for an infinite set of points in the plane that we introduced in connection to the Ginzburg–Landau model, and whose minimum is expected to be achieved by the "Abrikosov" triangular lattice. We obtain a next order asymptotic expansion of the partition function, and various characterizations of the behavior of the system at the microscopic scale. When the temperature tends to zero we show the system tends to "crystallize" to a minimizer of W.

I. M. Sigal: *Statics and Dynamics of Magnetic Vortices*

In this talk I will review recent results on existence, stability and dynamics of solutions of the Ginzburg–Landau equations of superconductivity exhibiting the vortex structure.

J. Sjöstrand: *Weyl Laws for Non-Self-Adjoint Differential Operators with Small Random Perturbations*

We describe some developments, tending to confirm the following general phenomenon: For elliptic differential operators on compact manifolds depending on a small random part, the eigenvalues distribute according to Weyl's law.

We also discuss a recent related result about the distribution of resonances near the real axis for Schrödinger operators.

A. Sobolev: *Discrete Spectrum Asymptotics for Certain Finite Band Lattice Operators*

Discrete spectrum asymptotics have been extensively studied for classical Jacobi matrices with the dominating growing diagonal part. We obtain analogous asymptotic formulas for multidimensional finite band lattice operators. The central idea of the method goes back to the Near Diagonalization Approach by G. Rozenblum, but the multi-dimensional nature of the problem leads to the occurrence of "resonant zones" in the lattice, which make the asymptotic properties of these operators more involved than in the one-dimensional case. The accurate description of the resonant zones is the main technical difficulty of this work.

J. P. Solovej: *Microscopic Derivation of the Ginzburg–Landau Model*

I will discuss how the Ginzburg–Landau model of superconductivity arises in an asymptotic limit of the microscopic Bardeen–Cooper–Schrieffer (BCS) model. The asymptotic limit may be seen to be of a semiclassical nature and one of the main difficulties is to derive semiclassical spectral asymptotics with minimal regularity assumptions. The talk will begin with a short introduction to a variational formulation of the BCS model. This is joint work with Frank, Hainzl, and Seiringer.

C. Tretter: *Spectral Inclusions for Operators with Spectral Gaps*

Analytical information about the spectra and resolvents of non-selfadjoint operators is of great importance for numerical analysis and applications. However, even for perturbations of selfadjoint operators there are only a few classical results. In this talk relatively bounded, not necessarily symmetric perturbations of selfadjoint operators with spectral gaps are considered. We present new spectral inclusion results and various modifications e.g. for gaps of the essential spectrum or for infinitely many gaps, and some applications.

W.-M. Wang: *Spectral Methods in PDE*

We develop a new geometric L^2-method for nonlinear PDE with applications to the energy supercritical NLS, NLW and some other equations.

T. Weidl: *Sharp Semiclassical Spectral Estimates with Remainder Terms*

I discuss recent developments on semiclassical spectral bounds on the eigenvalues of the Dirichlet Laplacian with sharp constants in the first order and additional remainder estimates of lower order as well as applications to heat kernel estimates and bounds for specific domains of infinite volume.
This includes joint work with H. Kovarik, S. Wugalter, A. Laptev and L. Geisinger.

S.-T. Yau: *Geometry and Spectrum of Graphs*