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PDE II
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Excercise Sheet 6 for 19. 06. 2017

Consider

$$E := \inf_{\substack{u\in H^1(\mathbb{R}^3)\\ \|u\|_2=1}} \left\{ \int_{\mathbb{R}^3} \left|\nabla u(x)\right|^2 \mathrm{d}x - \int_{\mathbb{R}^3} \frac{\left|u(x)\right|^2}{|x|} \mathrm{d}x + \frac{1}{2}\int_{\mathbb{R}^3} \left|u(x)\right|^4 \mathrm{d}x \right\}$$

6.1. Prove that $E \in (-\infty, 0)$.

6.2. Prove that there exists a minimizer $u_0 \geqslant 0$, on which E is attained.

6.3. Prove that u_0 satisfies

$$-\Delta u_0 - \frac{u_0}{|x|} + u_0^3 = \mu u_0$$

with some $\mu \in \mathbb{R}$.

6.4. Prove that $u_0 \in H^2(\mathbb{R}^3) \cap C^\infty\big(\mathbb{R}^3 \setminus \{0\}\big)$.