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PDE II
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Excercise Sheet 11 for 24. 07. 2017

Let $\Omega := B_1(0)$ be the unit ball in $\mathbb{R}^d$, $d \in \mathbb{N}$.

11.1. Prove that $u \in H_0^1(\Omega)$ if and only if the function

$$\widetilde{u}(x) := \begin{cases} u(x), & x \in \Omega, \\ 0, & x \notin \Omega \end{cases}$$

belongs to $H^1(\mathbb{R}^d)$.

11.2. Prove that

$$H_0^1(\Omega) = \big\{ f \in H^1(\Omega) : f|_{\partial\Omega} = 0 \text{ in } L^2(\partial\Omega) \big\}.$$

Note that "$\subseteq$" is already proved in the lecture, so it is "$\supseteq$" which needs a proof.

11.3. Prove that $u \in H_0^1(\Omega)$ if and only if there exists $C > 0$ such that

$$\left| \int_\Omega u(x) \nabla\varphi(x)\, \mathrm{d}x \right| \leqslant C \|\varphi\|_{L^2(\Omega)}$$

holds for all $\varphi \in H^1(\Omega)$.

Remark: The statements of Exercises 11.1 – 11.3 are valid for arbitrary Ω with $\partial\Omega \in C^1$.

11.4. Prove that for any $f \in L^2(0,1)$ there exists a unique solution $u \in H^2(0,1)$ for the boundary value problem

$$\begin{cases} -u'' + u = f, \\ u(0) = u(1), \\ u'(0) = u'(1). \end{cases}$$