

Excercises for the session on July 1

Exercise 1: In the proof of Lemma 9.6 (bosonic case): Let $V := \text{span}\{v_1, \tilde{v}_1\}$ and X as in the lecture notes. Prove that $\mathfrak{H} \oplus \mathfrak{H}^* = V \dot{+} X$ (direct sum).

Exercise 2: Provide (some of) the missing details in the proof of Theorem 9.9. For example:

- In the fermionic case:
 1. Prove that $\frac{1}{4} - \mathcal{A}_-^2$ is trace class.
 2. Prove (74) and that $0 \leq \lambda_i \leq 1/2$.
 3. Prove that $U^*\gamma U$, $V^*J\gamma J^*V$ and $U^*U\alpha\alpha^*$ are trace class.
 4. Prove that $\mathcal{A}_+ - 1/2$ is Hilbert–Schmidt.
- In the bosonic case:
 1. Is $C^2 - 1/4$ a Hilbert–Schmidt or trace class operator?
 2. Verify that C has an orthonormal eigenbasis.
 3. Prove that $\Gamma_\Psi \mathcal{S}(\Gamma_\Psi + \mathcal{S})$ has finite trace.
 4. Prove that the upper left entry of the matrix for $\mathcal{V}^*\Gamma\mathcal{V}$ is trace class.
 5. Verify that \mathcal{V} admits a unitary representation.