Mathematisches Institut der LMU
Prof. Dr. P. Müller
Dr. S. Morozov

Mathematical Quantum Mechanics II
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Homework 5

For Thursday, 1 June 2016

Throughout "operator" means a linear operator.

5.1. Let $\gamma \in [1, \infty]$. Prove that for any pair of self-adjoint operators H_0, H the property

$$(H_0 - z)^{-1} - (H - z)^{-1} \in \mathcal{T}^\gamma$$

does not depend on the choice of z from the intersection of the resolvent sets of H_0 and H.

5.2. Prove the following continuity property of generalized wave operators: Suppose that $W(H, H_0)$ exists. If $(A_n)_{n\in\mathbb{N}}$ is a sequence of self-adjoint trace class operators with $\|A_n\|_1 \underset{n\to\infty}{\longrightarrow} 0$, then $W_+(H + A_n, H_0)$ exists and converges to $W_+(H, H_0)$ strongly.

5.3. Let T be compact and R bounded operators in a separable Hilbert space $\mathcal{H}$. Prove that if TR and RT are trace class operators, then

$$\operatorname{tr} TR = \operatorname{tr} RT$$

holds.

5.4. The class of Hilbert-Schmidt operators consists of bounded operators T for which there exists an orthonormal basis $(g_l)_{l\in\mathbb{N}}$ such that

$$\|T\|_2^2 := \sum_{l\in\mathbb{N}} \|Tg_l\|^2 < \infty.$$

Prove that $\|T\|_2$ does not depend on the choice of the basis and that

$$\|T\|_2^2 := \sum_{k\in\mathbb{N}} s_k(T)^2$$

holds. In particular, the Hilbert-Schmidt class coincides with the class $\mathcal{T}^2$ of compact operators.