

Homework 4

For Thursday, 19 May 2016

4.1. Prove that for every $\varphi \in \mathcal{S}(\mathbb{R}^d)$ the map $t \mapsto e^{it(-\Delta)}\varphi$ is continuous as a function from \mathbb{R} to $L^p(\mathbb{R}^d)$ with $2 \leq p < \infty$ (cf. the proof of Theorem 3.3).

4.2. Prove that $(\mathcal{T}^1, \|\cdot\|_1)$ is a separable Banach space (in particular, $\|\cdot\|_1$ is a norm).

4.3. For $T \in \mathcal{T}^1$ and an ONB $\{g_l\}_l$ let

$$\operatorname{tr} T := \sum_l \langle g_l, T g_l \rangle.$$

Prove that $\operatorname{tr} T$ does not depend on the choice of $\{g_l\}_l$ and that tr is a bounded continuous functional on \mathcal{T}^1 with norm 1.