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Mathematical Quantum Mechanics II
SoSe 2016
21. 04. 2016

Homework 2

For Thursday, 28. April 2016

2.1. Prove the last two statements of Lemma 2.3(e): The wave operators $\Omega_\pm$ satisfy the intertwining relations

(a) $\Omega_\pm 1_A(H_0) = 1_A(H)\Omega_\pm$ for any Borel set $A \subset \mathbb{R}$ and

(b) $\Omega_\pm f(H_0) = f(H)\Omega_\pm$ for any bounded continuous function f on $\mathbb{R}$.

In the following $\mathcal{H}$ is a separable Hilbert space.

2.2. Prove that every bounded linear operator $T \in \mathcal{L}(\mathcal{H})$ admits a polar decomposition $T = W|T|$, with $|T| := (T^*T)^{1/2}$ non-negative and W isometric in $\mathcal{H}$.

2.3. Prove that every compact linear operator $T \in \mathcal{T}^\infty(\mathcal{H})$ admits a canonical representation (Schmidt expansion)

$$T = \sum_k s_k \langle \varphi_k, \cdot \rangle \psi_k,$$

where $\{\varphi_k\}$, $\{\psi_k\}$ are orthonormal systems complete in $\overline{T^*\mathcal{H}}$ and $\overline{T\mathcal{H}}$, respectively, and $\{s_k\}$ is the sequence of eigenvalues of $|T|$ in the nonincreasing order counted with multiplicities. Compute the Schmidt expansion of T^*.

Hint: Use the spectral theorem for $|T|$.