Program Extraction with Nested Inductive/Coinductive Definitions

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Abstract

We present our work on program extraction and a case study on uniformly continuous functions working in our proof system Minlog [4].

Ulrich Berger and Monika Seisenberger [1,2] inductively/coinductively defined a predicate of the uniform continuity and informally extracted Haskell programs from their constructive proofs of it. Our work enriches the Theory of Computable Functionals [3] and its computer implementation Minlog in order to formalize case studies by Berger and Seisenberger.

We extract from formal proofs programs which translate a uniformly continuous function on Cauchy reals in $[-1, 1]$ into a non-well founded tree representation, and vice versa. Via Kreisel's modified realizability interpretation, the extracted programs involve certain recursion and corecursion operators which come from nested inductive/coinductive definitions. The non-well founded tree representation of uniformly continuous functions is of ground type. In this way, we manage to understand uniformly continuous functions through approximating non-well founded objects.

References

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* Kenji Miyamoto is supported by the Marie Curie Initial Training Network in Mathematical Logic – MALOA – From MAthematical LOgic to Applications, PITN-GA-2009-238381