

Mathematical Quantum Mechanics

TMP Programme, Munich – Winter Term 2010/2011

|| *These topics that shall be discussed together in the first week exercise/tutorial sessions of the course (for in the zero-th week no exercise worksheet was issued), i.e., in the sessions of 26, 27, and 28 October.* ||

A. Why do we postulate that the space of states of a quantum system has to be a (complex) Hilbert space? Which features of a Hilbert space (linearity, dimensionality, underlying complex field, geometric structure, completeness) are dictated from Physics and how? Why do we need completeness, in particular? Why do we need a wave-function to be a Lebesgue, and not just a Riemann square integrable function?

B. In class you learnt that classically atoms are unstable, for the Hamilton function can take arbitrarily large and negative values and therefore electrons eventually collapse onto the nucleus. Nevertheless, atoms could appear stable for any practical purposes, for it might take twice as the age of the universe for an electron rotating around a nucleus to fall onto it. With the laws of classical electromagnetism, compute the time that the electron of an Hydrogen atom would take classically to fall onto the nucleus by losing its energy by radiation (hint: Larmor's formula).

C. Recap on Fourier transform. Its definition and action on L^p spaces. Fourier transform as a unitary map on wave-functions. Duality among position and momentum operators via Fourier transform.

D. Recap on the degeneracy structure of the Hydrogen eigenvalues. Scaling of the sum of the first N eigenvalues.

E. Symmetric operators have real expectations (and vice versa) and eigenvectors corresponding to distinct eigenvalues are orthogonal. Equivalent ways to exponentiate bounded symmetric operators (see exercises 3.1 and 3.2 of the handout "*Basic concepts in QM*") and what shall be exported to the unbounded case.

F. Unitary equivalence on Hilbert spaces. Gauge invariance of the kinetic energy operator in the presence of an external magnetic field (see exercise 4.1 of the handout "*Basic concepts in QM*")