Christmas Problems

Dec 21, 2004

These two problems are only for fun, you can think about them during the Christmas break. If you have a solution, let me know (either verbally or in written form). The best solutions can be presented in class after the break.

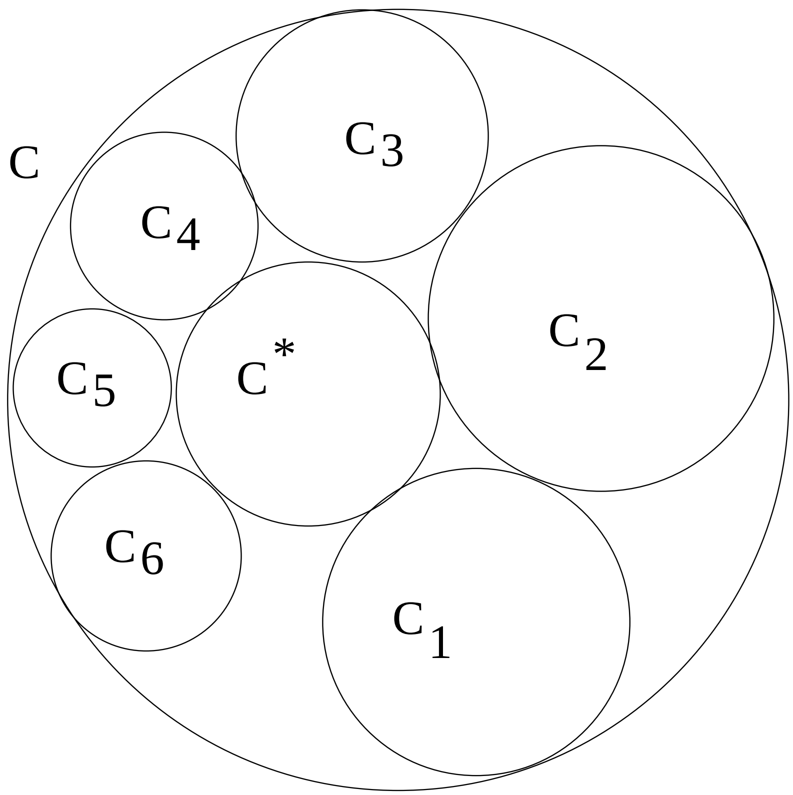
Problem 1. Given n non-collinear points in the plane. Show that there exist a line that contains exactly two of these points.

Problem 2. Given two circles, C and C^* in the plane such that C^* lies "inside" of C and they do not touch each other. Consider a sequence of circles, $C_1, C_2, \ldots$ that are drawn "in between" C and C^* such that each C_j touches C from inside and touches C^* from outside, moreover, C_j touches both C_{j-1} and C_{j+1} from outside.

After a few of these circles, the chain of circles goes "around" in the "ring" between C and C^*. Note that once the initial circle C_1 is fixed, $C_2, C_3, \ldots$ are uniquely determined. (See picture on the next side)

Suppose that for some initial C_1 we find that C_n exactly touches C_1, i.e. the chain exactly closes. Prove that then for any initial C_1 the generated chain will close.

[Hint: Inversion]


C
C3
C4
C2
C5
C*
C6
C1