

The Atiyah-Singer index theorem

SEMINAR SOSEM 2024

as of February 1, 2024

The Atiyah-Singer index theorem establishes a deep connection between geometric, topological and analytic properties of manifolds. It subsumes well-known results such as the Gauß-Bonnet-Chern, the Riemann-Roch-Hirzebruch and the Hirzebruch signature theorem as special cases and puts them in a common conceptual framework.

Important geometrically defined differential operators on Riemannian manifolds such as the Laplace operator and Dirac operators belong to the class of elliptic operators. The index of an elliptic operator on a closed manifold is defined as the difference of the dimensions of its kernel and cokernel, and hence is an *analytic* quantity. The index only depends on the “topological type” of the symbol of the operator (which comprises the “highest order” terms) and the index theorem expresses it in terms of *topological* quantities, namely by certain combinations of characteristic numbers. These in turn can be represented as integrals of curvature quantities, which yields the connection to *geometry*. The simplest example of this kind of relation between topology and geometry is the Gauß-Bonnet theorem for surfaces.

References:

J. Roe, *Elliptic operators, topology and asymptotic methods*, 2nd ed., Pitman
Research Notes in Math. 395, Addison Wesley Longman, 1998

N. Berline, E. Getzler, M. Vergne, *Heat Kernels and Dirac Operators*, Springer, 1992

H.B. Lawson, M.-L. Michelsohn, *Spin Geometry*, Princeton University Press, 1989

Prerequisites: basic notions of differential geometry as covered e.g. in the class on differentiable manifolds

For students of mathematics or physics (master, TMP)

Time: Thursday 16-18 in room B 252 (tentative)

First meeting and distribution of talks: Thursday **april 18** at **16:15h**

Registration: If you are interested in participating, please email me (preferably before the first meeting).