

# ALEA in Europe Meeting 2016

## Program and Abstracts

February 22-26, 2016

University of Munich, Institute for Mathematics  
Theresienstr. 39, D-80333 München

### Program

**Location:** all talks will take place in room B005. Coffee and lunch breaks (on Monday and Wednesday) will take place in room B046. Both rooms are located on the ground floor of the building at Theresienstr. 39.

	<b>Monday</b>	<b>Tuesday</b>	<b>Wednesday</b>	<b>Thursday</b>	<b>Friday</b>
8:30 – 9:15	Registration				
9:15 – 10:45	Christina Goldschmidt	Marc Noy	Marc Noy	Marc Noy	Christina Goldschmidt
10:45 – 11:30	Coffee	Coffee	Coffee	Coffee	Coffee
11:30 – 12:30	Johannes Lengler	Mihyun Kang	Carola Doerr	Benedikt Stuffer	Matthias Schacht
12:30 – 13:30	Lunch (B046)	Lunch (participants on their own)	Lunch (B046)	Lunch (participants on their own)	
13:30 – 15:00	Benjamin Doerr	Christina Goldschmidt	Excursion	Benjamin Doerr	
15:00 – 15:30	Coffee	Coffee		Coffee	
15:30 – 17:00	Short talks	Benjamin Doerr		Short Talks	

**Excursion:** on Wednesday afternoon an excursion will be organized for the participants. You can choose between a guided tour to the Pinakothek of Modern Art and a guided tour to the Egyptian museum, which are located close to the institute.

## Lecture Series

### Christina Goldschmidt – Scaling Limits of Random Discrete Trees

In the last 30 years, random combinatorial structures and their limits have been a flourishing area of research at the interface between probability and combinatorics. In this course, I hope to show you some of the beautiful theory that arises when considering scaling limits of random trees.

Trees are fundamental objects in combinatorics and the enumeration of different classes of trees is a classical subject. We will take as our basic object the genealogical tree of a Galton-Watson branching process. (As well as having nice probabilistic properties, this class turns out to include various natural types of random combinatorial tree in disguise.) In the same way as Brownian motion is the universal scaling limit for centered random walks of finite step-size variance, it turns out that all critical Galton-Watson trees with finite offspring variance have a universal scaling limit, Aldous' Brownian continuum random tree (CRT). (In fact, this is not just an analogy!) The CRT is a so-called R-tree, a path metric space which is connected and has no cycles. In the first lecture, I will discuss the convergence to the CRT (and what exactly we mean by convergence of a tree). Then I will move on to consider what happens in the infinite variance case, where different objects (the stable trees, introduced by Duquesne, Le Gall and Le Jan) arise in the limit. In the final part (time-permitting) I will talk about some interesting characterisations of the limit objects.

### Benjamin Doerr – Epidemic and Gossip-Based Algorithms

Epidemic or gossip-based algorithms – the two mean nearly the same – are distributed randomized algorithms in which all communication is done via nodes calling random neighbors. Despite this unorganized approach, such algorithms can be surprisingly powerful. A simple example is the randomized phone chain (or the randomized rumor spreading algorithm in computer science language), which makes a piece of information known in a group of people by the simple mechanism that in each round of the protocol, each informed member of the group calls a random other one. Contrary to most phone chains used in practice, this mechanism needs only a logarithmic number of rounds to communicate a piece of information to everyone. At the same time, it is highly robust. If each call gets lost independently with 50% probability, then the rumor spreading time increases only by 82%.

In this small series of lectures, I will give an introduction to this young algorithmic area, which draws its attractiveness from the combination of mathematical beauty and recent real-world challenges, e.g., communication and algorithmic tasks in wireless sensor networks or mobile ad-hoc networks.

### Marc Noy – Random Graphs from Constrained Classes

We discuss the problem of analyzing properties of random graphs from a class given by a global constraint, such as being  $d$ -regular, triangle-free or acyclic.

The first lecture will consist of an overview, covering different approaches and methods. The second and third lectures will focus on classes of graphs defined by excluded minors using, respectively, combinatorial and analytic methods.

## Invited Talks

### Spotlight on the Analysis of Evolutionary Algorithms

*Carola Doerr (Université Pierre et Marie Curie - Paris 6)*

Evolutionary algorithms (EAs) form a powerful class of optimization techniques for both industrial and academic applications. A large research community exists that studies EAs from a scientific angle. In this talk, we show that EAs and other randomized search heuristics are also interesting from a mathematical point of view. We provide an example showing how running time analysis and complexity theory have inspired the development of a novel algorithm which provably outperforms existing techniques in evolutionary computation. Our example also affirmatively answers one of the main open questions in EA theory, namely the usefulness of crossover (i.e., the recombination of two or more search points) on simple fitness landscapes.

### Sparse Random Graphs on Surfaces

*Mihyun Kang (TU Graz)*

Graphs on a 2-dimensional surface have been among the most studied combinatorial objects in graph theory, enumerative combinatorics, and statistical physics. In this talk we discuss recent results on sparse random graphs on orientable surfaces. Key techniques include the constructive decomposition of graphs along connectivity, methods from analytic combinatorics, and the core-kernel method as well as combinatorial counting arguments.

### Modelling Social Networks – For Dummies

*Johannes Lengler (ETH Zurich)*

There are many models for social and technological networks, none of which is completely satisfying. Classical models like Preferential Attachment graphs or Chung-Lu random graphs are well-understood, but they fail to have some crucial properties observed in real-world networks (e.g., that vertices tend to form clusters). Some newer alternatives, most notably the celebrated Hyperbolic Random Graphs, have better properties, but they are typically very technical and hard to understand.

I will present a new framework for social and technological networks, so-called Geometric Inhomogeneous Random Graphs (GIRGs). The model generalizes many previous models (including hyperbolic random graphs), but it is nevertheless much simpler. On top, it is very versatile: for example, imagine you want to describe an individual in your social network by her living place and her field of work, and two nodes should have a good chance to know each other if either they have a very similar field of work (e.g., probabilistic graph theory) or they have a somewhat related field of work (e.g., math) but live in the same town. The framework will allow you to build a random graph model with such properties in five minutes, and it will guarantee that all basic properties of social networks are satisfied.

In the talk I will review the different properties of the framework. Many of the presented results are new even for special cases like hyperbolic random graphs. No background knowledge on random graph models is required. All results are joint work with Karl Bringmann and Ralph Keusch.

## **Extremal Combinatorics in Random Discrete Structures**

*Mathias Schacht (University of Hamburg)*

We survey on recent results at the intersection of extremal combinatorics and random graph theory. More precisely, we consider thresholds for extremal properties of random discrete structures. Among other problems, we shall discuss the threshold for Szemerédi's theorem on arithmetic progressions in random subsets of the integers and the threshold for Turan-type problems for random graphs and hypergraphs, which were obtained independently by Conlon and Gowers and by the speaker. Furthermore, we discuss recent general results on independent sets in hypergraphs by Balogh, Morris and Samotij and by Thomason and Saxton, which led to new proofs of these results and already had many other applications in the area.

## **Limits of Random Tree-like Combinatorial Structures**

*Benedikt Stufler (University of Munich)*

Using a unified approach, we obtain scaling limits and local weak limits, in particular Benjamini-Schramm limits, of a variety of random tree-like combinatorial structures. In particular, we discuss limits of random graphs from subcritical classes in the labelled and unlabelled setting and random planar maps from classes of simple outerplanar maps, sampled according to Boltzmann weights, which encompasses the case of uniform graphs or maps from such classes. Our approach also yields applications to "less tree-like" objects, such as random planar graphs, however without establishing limits.

## Short Talks

### Limit Laws of Vertex Degree Distribution in Planar Maps

*Gwendal Collet (TU Vienna)*

We consider the family of rooted planar maps where the vertex degrees belong to a (possibly infinite) set  $D$  of positive integers. Using bijective and analytic tools, we first recover the universal asymptotic behaviour of planar maps, and furthermore we establish that the number of vertices of degree  $d \in D$  satisfies a central limit theorem.

### Bootstrap Percolation in Directed and Inhomogeneous Random Graphs

*Nils Detering (University of Munich)*

Bootstrap percolation is a process that is used to model the spread of an infection on a given graph. In the model considered here each vertex is equipped with an individual threshold. As soon as the number of infected neighbors exceeds that threshold, the vertex gets infected as well and remains so forever. In this paper we perform a thorough analysis of bootstrap percolation on a novel model of directed and inhomogeneous random graphs, where the distribution of the edges is specified by assigning two distinct weights to each vertex, describing the tendency of it to receive edges from or to send edges to other vertices. Under the assumption that the limiting degree distribution of the graph is integrable we determine the typical fraction of infected vertices. Our model allows us to study settings that were outside the reach of current methods, in particular the prominent case in which the degree distribution has an unbounded variance. Among other results, we quantify the notion of "systemic risk", that is, to what extent local adverse shocks can propagate to large parts of the graph through a cascade, and discover novel features that make graphs prone/resilient to initially small infections.

### New Bounds for the Chromatic Number of Dense Random Graphs

*Annika Heckel (University of Oxford)*

The chromatic number  $\chi(G)$  of a graph  $G$ , defined as the minimum number of colours required to colour the vertices so that no two adjacent vertices are coloured the same, is one of the central concepts in graph theory. The chromatic number of the random graph  $G_{n,p}$ , where each of the  $\binom{n}{2}$  potential edges between  $n$  vertices is present independently with probability  $p$ , has been the subject of intense study since at least the 1970s.

A celebrated breakthrough by Bollobás in 1987 first established the asymptotic value of  $\chi(G_{n,p})$  for constant  $p$ , and a considerable amount of effort has since been spent on refining Bollobás' approach, resulting in increasingly accurate bounds. Despite this, up until now there has been a gap of size  $O(1)$  in the denominator between the best known upper and lower bounds.

In this talk, new bounds for  $\chi(G_{n,p})$  will be presented which match each other up to a term of size  $o(1)$  in the denominator. In particular, they narrow down the optimal colouring rate, defined as the average colour class size in a colouring with the minimum number of colours, to an interval of length  $o(1)$ . These bounds were obtained through a careful application of the

second moment method rather than a variant of Bollobás' method. Somewhat surprisingly, the behaviour of  $\chi(G_{n,p})$  is different for  $p \leq 1 - 1/e^2$  and for  $p > 1 - 1/e^2$ , with a different limiting effect being dominant in each case.

## **Jigsaw Percolation on Random Hypergraphs**

*Christoph Koch (TU Graz)*

Jigsaw percolation was introduced by Brummit, Chatterjee, Dey, and Sivakoff as a model for interactions within a social network. It was inspired by the idea of collectively solving a puzzle. The premise is that each individual of a group of people has a piece of a puzzle all of which must be combined in a certain way to solve the puzzle. The compatibility of different puzzle pieces and the information which pairs of people meet are stored in two graphs (on a common vertex set), the puzzle graph and the people graph. Bollobás, Riordan, Slivken, and Smith studied the process when both graphs are given by independent binomial random graphs. In the analysis, a crucial role is played by sets of vertices which are “internally spanned”, a property that is stronger than being connected in both graphs independently. We generalise the process and this notion to hypergraphs, for a whole family of possible definitions of connectivity. We provide the asymptotic order of the critical threshold probability for percolation when both hypergraphs are chosen binomially at random, extending the result of Bollobás, Riordan, Slivken, and Smith.

This is joint work with Béla Bollobás, Oliver Cooley, and Mihyun Kang.

## **Rectangular Young Tableaux**

*Philippe Marchal (Université Paris 13)*

The limit shape of the random surface associated with a rectangular Young tableau has been determined by Pittel and Romik (2007). We study the fluctuations of this limit shape on the edge of the tableau. We show that these fluctuations are gaussian in the corner and Tracy-Widom on the edge away from the corner.

## **Leader Election Algorithms**

*Vlady Ravelomanana (Université de Paris Nord)*

Leader elections and size approximation of unknown networks are fundamental primitives that have found applications in many areas of computer science, namely in distributed computing. In this talk, we first review existing protocols in the radio networks context with no collision detection. Then we will design energy-efficient algorithms for electing a leader and approximating the network size in  $O(\log n)$  mean time slots with no station being awake for more than  $O(\log \log n)$  rounds. Note that in the same context, all previous known algorithms that, with significant probability or in expectation compute an approximate size of the networks worked in time much larger than  $O(\log n)$ .