

## Exercises for Stochastic Processes

Let  $(B_t)_{t \geq 0}$  be the standard Brownian motion and  $(\mathfrak{F}_t)_{t \geq 0}$  the right-continuous completion of the natural filtration. In the following, the notion of stopping time is used with respect to  $(\mathfrak{F}_t)_{t \geq 0}$ .

1. Let for  $\tau := \inf \{t \geq 0 \mid B_t \in \{a, b\}\}$  for  $a < 0 < b$ .
  - (a) Determine the distribution of  $B_\tau$  and compute the expected value of  $\tau$ .
  - (b) Compute  $\mathbb{E}[\tau^2]$ .
2. Show that there exists a stopping time  $\tau$  with  $\mathbb{E}[\tau] = \infty$  and  $\mathbb{E}[B_\tau^2] < \infty$ .
3. Find stopping times  $\sigma$  and  $\tau$  with  $\mathbb{E}[\sigma] < \infty$ ,  $\sigma \leq \tau$  almost surely and

$$\mathbb{E}[B_\sigma^2] > \mathbb{E}[B_\tau^2].$$

4. Show that

$$\mathbb{E}[\tau^2] \leq 4\mathbb{E}[B_\tau^4]$$

for a stopping time  $\tau$  with  $\mathbb{E}[\tau] < \infty$ .

**Deadline:** Tuesday, 18.12.2018. Hand in in groups, please!