Mathematisches Institut der Universität München
Prof. Sabine Jansen, Leonid Kolesnikov

WS 18/19
Sheet 3

Exercises for Stochastic Processes

1. Let B be a Brownian motion. Show that the *Brownian bridge*, defined by
$$X_t := B_t - tB_1, \quad t \in [0,1],$$
is a Gaussian process. Compute its covariance kernel.

2. Let B be a Brownian motion.
   (a) Show that
$$\lim_{t\to\infty} \frac{B_t}{t} = 0 \quad \text{a.s.}$$
   (b) Show that
$$\limsup_{t\uparrow\infty} \frac{B_t}{\sqrt{t}} = \limsup_{t\downarrow 0} \frac{B_t}{\sqrt{t}} = \infty$$
   and
$$\liminf_{t\uparrow\infty} \frac{B_t}{\sqrt{t}} = \liminf_{t\downarrow 0} \frac{B_t}{\sqrt{t}} = -\infty \quad \text{a.s.}$$

3. Let X be a continuous-time, real-valued process with independent increments. Show that X satisfies the (simple) Markov property as defined in the lecture.

Deadline: Tuesday, 13.11.2018. Hand in in groups, please!