Exercises for Stochastic Processes

1. Let X be an irreducible Markov chain with standard transition function p_t. Show that X is recurrent (i.e., all its states are recurrent) if a recurrent state exists.

2. Let X be a Markov chain with transition function p_t and starting distribution π. Show that X is stationary if and only if π is invariant with respect to p_t.

3. Let X be a Markov chain on a finite set E with standard transition function p_t and weakly conservative Q-matrix Q. Let π be a strictly positive measure on E. Show that π is reversible if and only if

$$\pi(x)q(x,y) = \pi(y)q(y,x) \quad \forall x,y \in E.$$

4. Let $p \in (0,1)\backslash\{\frac{1}{2}\}$. Find two invariant measures for the asymmetric random walk on $\mathbb{Z}$ defined by the Q-matrix

$$q(x,x-1) = 1-p, \quad q(x,x) = -1, \quad q(x,x+1) = p, \qquad \forall x \in \mathbb{Z},$$

such that both measures are not multiples of each other.

Hint: For the Poisson distribution we have the following estimate:

$$\mathbb{P}(\mathrm{POI}(\lambda) \geq k) \leq \exp\left(-k\log\left(\frac{k}{\lambda e}\right) + \lambda\right).$$

Use this estimate to find a function g such that $\sum \pi(x)g(x) < \infty$ and $\mid \frac{d}{dt}P_t(x,y) \mid \leq g(x)$ for every $x,y \in \mathbb{Z}$.

Deadline: Tuesday, 22.01.2019. Hand in in groups, please!