Automatic sequences: between order and chaos

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Between "ordered sequences" (e.g., periodic ones) and "chaotic sequences" (e.g., random sequences) there is some room for intermediate sequences that are neither periodic nor really "random". Among these neither/nor sequences, a class called "automatic sequences" has many interesting properties and occurs in several distinct fields in mathematics, computer science, or physics.

Let us write down the (beginning of the) most famous automatic sequence, namely the "Prouhet-Thue-Morse" sequence. Can we immediately see patterns in it?

$$0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ \ldots$$

We will propose a "promenade" among automatic sequences showing some of their properties in number theory (transcendency of numbers and of series), in harmonic analysis (deterministic sequences having the same behavior as "random" sequences), in theoretical computer science (repetitions in infinite words, Hanoi tower, finite automata, cellular automata), and in physics (one dimensional Ising model, quasicrystals). A brief allusion to the use of automatic sequences in music will conclude the talk.