

# Seminar Geometric Group Theory

## SUMMER SEMESTER 2024

The idea behind *geometric group theory* is to study infinite, finitely generated groups using methods from geometry and topology. This often allows to prove strong theorems in group theory using very clear and flexible arguments. Beyond reproving classical group theory results in a clearer way, it leads to vast generalisations. In this seminar, we will explore some more modern aspects of this extremely active subfield of maths.

This seminar is planned as a continuation as the geometric group theory lecture from the winter semester 2023/24. Goals include

- More results on hyperbolic groups (e.g. contractability of the Rips complex, and algebraic finiteness consequences),
- Boundaries of hyperbolic spaces,
- More algorithmic aspects of groups,
- An introduction to automorphism groups of free groups,
- Geometry of topologically motivated groups (e.g. braid groups),
- consequences of actions on hyperbolic spaces.

Topics can be adapted to the interest of the audience.

This seminar is *mainly* aimed at students with a background in basic geometric group theory (e.g. as a continuation of my course from the winter semester). However, since some of the talks will introduce interesting examples or standalone topics, the seminar is accessible also to students who have not participated in the lecture, but are interested in geometric group theory.

### Literature

- (1) John Meier, “Groups, Graphs and Trees: An Introduction to the Geometry of Infinite Groups”, Cambridge University Press, 2008.
- (2) Clara Löh, “Geometric Group Theory: An Introduction”, Springer, 2017.
- (3) Matt Clay and Dan Margalit, “Office Hours with a Geometric Group Theorist”, Princeton Press, 2017.
- (4) Cornelia Drutu and Michael Kapovich, “Geometric Group Theory”

**For:** Students of Mathematics (Master); TMP students with an interest in pure mathematics.

**Questions and Sign-Up** via email to: [hensel@math.lmu.de](mailto:hensel@math.lmu.de)

## Information on Talks

Below are some possible topics for talks.

- (1) **Groups in nature I:** Reflection Groups (Chapter 2 of Meier), RAAGs, braid groups
- (2) **The word problem and languages** (Topics from Meier Chapters 5 and 7)
- (3) **Unsolvable word problem** Construction of a group with unsolvable word problem (following e.g. Simpson, “A slick proof of the unsolvability of the word problem for finitely presented groups”)
- (4) **The Rips Complex (is contractible)** Constructing a contractible space on which a hyperbolic group acts properly discontinuously
- (5) **Centralisers in hyperbolic groups** are undistorted, and consequences (no free Abelian subgroups of rank at least 2).
- (6) **The boundary at infinity I** of a hyperbolic space: construction, various interpretations
- (7) **The boundary at infinity II** of a hyperbolic space: quasi-isometric invariance, connection to ends.
- (8) **Groups in nature II:** Free Groups and their automorphisms in topology (Chapter 3 of Meier, and more), Mapping Class Groups (Farb-Margalit) and their non-hyperbolicity
- (9) **Hyperbolicity Criteria** Proving hyperbolicity by “guessing geodesics”
- (10) **Hyperbolic spaces for non-hyperbolic groups**
- (11) **Consequences of actions on hyperbolic spaces**