Winter term 2021

Prof. D. Kotschick
S. Gritschacher

Mathematical Gauge Theory II

Sheet 10

Exercise 1. Let (X, J) be a closed connected almost complex 4-manifold, and $\mathfrak{s}_L$ the canonical Spin^c-structure $\mathfrak{s}_{\text{can}}$ twisted by a line bundle L. Show that for generic parameters the expected dimension of the moduli space of irreducible solutions is

$$\dim(\mathcal{M}^*_\omega) = \langle c_2((V_L)_+), [X]\rangle = \langle c_1(L)^2, [X]\rangle + \langle c_1(L)c_1(TX), [X]\rangle.$$

Exercise 2. (The even expected dimension case) Let (X, g) be a smooth closed oriented Riemannian 4-manifold endowed with a Spin^c-structure $\mathfrak{s}$. Show that if the expected dimension of the moduli space is even, then $b_2^+(X) - b_1(X)$ is odd.

Exercise 3. (Seiberg-Witten equations on $S^2 \times S^2$) Consider $S^2 \times S^2$ with the product metric, where each factor is a round sphere, i.e. of constant curvature.

(a) Determine the moduli spaces of solutions to the unperturbed Seiberg-Witten equations for all Spin^c-structures.

(b) Conclude that whenever the moduli space is non-empty, then the expected dimension is negative.

Exercise 4. (Seiberg-Witten equations on $\#n(S^1 \times S^3)$) Consider $S^1 \times S^3$, with the product metric coming from two round factors.

(a) Show that there is a unique Spin^c-structure, and determine the moduli space of solutions to the unperturbed Seiberg-Witten equations. How does the dimension of the result compare to the expected dimension?

(b) Extend this discussion to connected sums of several copies of $S^1 \times S^3$.
[Hint: you can use the fact that the connected sum of two Riemannian manifolds with positive scalar curvature admits a metric with positive scalar curvature.]

You can hand in solutions in the lecture on Thursday, 27 January 2022.