LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Winter term 2021

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Mathematical Gauge Theory II

Sheet 7

Exercise 1. (The quadratic form σ and charge conjugation) We define charge conjugation on spinors in $V_+ \cong \mathbb{C}^2$ as:

$$J\colon V_+ \longrightarrow V_+$$
$$\begin{pmatrix} a \\ b \end{pmatrix} \longmapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}.$$

Prove that

$$\sigma(J\Phi, J\Phi) = -\sigma(\Phi, \Phi) \quad \forall \Phi \in V_+.$$

Exercise 2. (Rescaling the metric) Let (X, g) be a smooth closed oriented Riemannian 4-manifold with a Spin^c-structure $\mathfrak{s}$ and let $\omega \in \Omega^2_+(X, i\mathbb{R})$. Consider the rescaled metric $\widetilde{g} = \lambda^2 g$ for $\lambda \in \mathbb{R}^+$. Prove that if (A, Φ) is a solution to the ω-perturbed Seiberg-Witten equations on (X, g), then $(A, \lambda^{-1}\Phi)$ satisfies the Seiberg-Witten equations on $(X, \widetilde{g})$. [Notice that the Clifford module $\gamma\colon TX \longrightarrow \mathrm{End}(V)$ is rescaled.]

Exercise 3. (Reducible solutions I) Let (X, g) be a smooth closed oriented Riemannian 4-manifold endowed with a Spin^c-structure $\mathfrak{s}$. Show that the unperturbed Seiberg-Witten equations admit a reducible solution if and only if the harmonic representative of $c_1(L_{\mathfrak{s}})$ is anti-self-dual.

Exercise 4. (Reducible solutions II) Let (X, g) be a smooth closed oriented Riemannian 4-manifold endowed with a Spin^c-structure $\mathfrak{s}$. Show that if $b_2^+(X) > 0$ and $c_1(L_{\mathfrak{s}}) \neq 0$, then for a generic metric on X the unperturbed Seiberg-Witten equations do not admit reducible solutions.

You can hand in solutions in the lecture on Thursday, 9 December 2021.