LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Summer term 2015
June 8, 2015

Functional Analysis
Exercise Sheet 8

Reflexive spaces, Banach adjoint, and compact operators

- *First version deadline:* **June 15** (13:30). *Final hand in deadline:* **June 29** (13:30)

Exercise 1 (5 points). *Prove that ℓ^p is reflexive for $1 < p < \infty$ and that ℓ^1 is not reflexive.*

Exercise 2 (5 points). *Let X be a reflexive Banach space. Prove that for each $\varphi \in X^*$ there exists $x \in X$ with $\|x\| = 1$ such that $|\varphi(x)| = \|\varphi\|$.*

Exercise 3 (5 points). *Suppose that U is a dense subspace of a normed space X. Prove:*

(*i*) *A bounded linear map on X is uniquely determined by its action on U.*

(*ii*) *U^* and X^* are isometrically isomorphic.*

Exercise 4 (5 points). *Consider $S : \ell^1 \to \ell^2, S(x_1, x_2, \dots) = (0, x_1, x_2, \dots)$. Prove that this defines a bounded linear map and find an operator $T : \ell^2 \to \ell^\infty$ such that $S' = \phi_1 T \phi_2^{-1}$, where S' denotes the Banach conjugate/adjoint of S, and $\phi_1 : \ell^\infty \to (\ell^1)^*$, $\phi_2 : \ell^2 \to (\ell^2)^*$ are the isomorphisms given by the dual pairing $\langle \cdot, \cdot \rangle$. Compute $\|S\|$ and $\|S'\|$.*

Exercise 5 (5 points). *Let X and Y be normed spaces. Decide which of the following operators are compact (and prove your claim):*

(*i*) $T : C[0,1] \to C[0,1],\ Tf(x) = f(0) + xf(1)$.

(*ii*) $id : X \to X,\ x \mapsto x$.

(*iii*) $F \in \mathcal{L}(X, Y)$ *with* $\dim \operatorname{Ran}(F) < \infty$.

For general informations please visit http://www.math.lmu.de/~gottwald/15FA/