

ADVANCED ANALYSIS – WiSe 2019/20

Exercise sheet 7

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Emanuela Giacomelli – emanuela.giacomelli@math.lmu.de

Exercise 1. [20 points]

Consider the function $f(x) = |x|^{-n}$ on \mathbb{R}^n . Although this function is not in $L^1_{\text{loc}}(\mathbb{R}^n)$, it is defined as a distribution for test functions that vanish at the origin, by

$$T_f(\phi) = \int_{\mathbb{R}^n} |x|^{-n} \phi(x) dx.$$

- Show that there is a distribution $T \in \mathcal{D}'(\mathbb{R}^n)$ that agrees with T_f for functions that vanish at the origin. Give an explicit formula for one such T .
- Characterize all such T 's. (Hint: Theorem 6.14 from Lieb Loss may be helpful here).

Exercise 2. [20 points]

Show that if Ω is connected and if $T \in \mathcal{D}'(\Omega)$ has the property that $D^\alpha T = 0$ for all $|\alpha| = m + 1$, then T is a multinomial of degree at most m , i.e., $T = \sum_{|\alpha| \leq m} C_\alpha x^\alpha$.