

ADVANCED ANALYSIS – WiSe 2019/20

Exercise sheet 6

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Exercise 1. [15 points]

Let \mathcal{N}_T be the null space of a distribution, T . Show that there is a function $\phi_0 \in \mathcal{D}$ so that every element $\phi \in \mathcal{D}$ can be written as $\phi = \lambda\phi_0 + \psi$ with $\psi \in \mathcal{N}_T$ and $\lambda \in \mathbb{C}$.

Exercise 2. [15 points]

Show that a function f is in $W^{1,\infty}(\Omega)$ if and only if $f = g$ a.e. where g is a function that is bounded and Lipschitz continuous on Ω , i.e., there exists a constant C such that

$$|g(x) - g(y)| \leq C|x - y| \text{ for all } x, y \in \Omega.$$

Exercise 3. [10 points]

- Let $\Omega = (-1, 1)$ and let $f(x) = |x|$, prove that f' exists in the weak sense and calculate it.
- Let $\Omega = (-1, 1)$ and let $g(x) = \operatorname{sgn}(x)$. Does g' exist in the weak sense? If so, compute it.
- Let $h(x) := \operatorname{sgn}(x_1) + \operatorname{sgn}(x_2)$ in $\Omega = (-1, 1)^2$. Do $D_{12}h$ and $D_{21}h$ exist in the weak sense? If so, compute them.