

ADVANCED ANALYSIS – WiSe 2019/20

Exercise sheet 9

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Exercise 1. [15 points]

Adapt proof of Theorem 8.6 (in the book *Analysis* by Lieb and Loss) in the case $n = 2$.

Exercise 2. [15 points]

Let

$$\mathcal{E}(\psi) := \int |\nabla\psi|^2 - \int |\psi|^2 \frac{1}{|x|^{2-\epsilon}}$$

Prove that

$$E_0 = \inf \{ \mathcal{E}(\psi) : \psi \in H^1(\mathbb{R}^3), \|\psi\|_2 = 1 \} > -\infty$$

Exercise 3. [10 points]

Show that the following inequality

$$\|\nabla f\|_p \geq C_{p,n} \|f\|_q,$$

cannot be true if $q \neq \frac{np}{n-p}$.